Newmark's Method

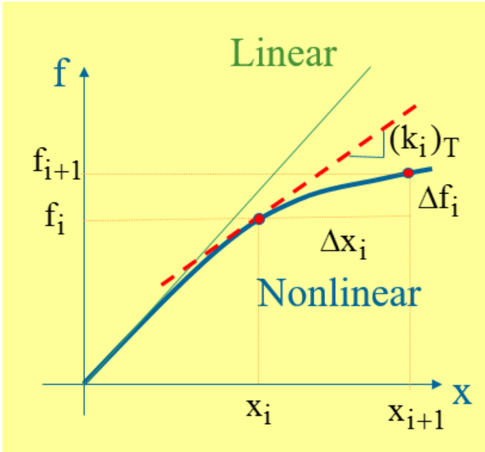
for nonlinear systems

EOM of nonlinear system

$$m\ddot{x}(t)+c\dot{x}(t)+f(x)=p(t)$$

$$m\ddot{x}_i(t)+c\dot{x}_i(t)+f(x_i)=p_i$$

$$m\ddot{x}_{i+1}(t)+c\dot{x}_{i+1}(t)+f(x_{i+1})=p_{i+1}$$



$$m\Delta\ddot{x}_i+c\Delta\dot{x}_i+\Delta f_i=\Delta p_i$$

$$\Delta f_i=f(x_{i+1})-f(x_i)$$

$$=\frac{\Delta f_i}{\Delta x_i}\Delta x_i\cong(k_i)_T\,\Delta x_i$$

Tangential stiffness

function of displacement

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$$m\Delta\ddot{x}_i(t)+c\Delta\dot{x}_i(t)+(k_i)_T\Delta x_i=\Delta p_i$$

Newmark's Scheme $\longrightarrow$ $\Delta x_i \longrightarrow x_{i+1},\ \dot{x}_{i+1},\ \ddot{x}_{i+1}$

Correction (minimize the unbalanced force)

Original $\quad m\Delta\ddot{x}_i(t)+c\Delta\dot{x}_i+\Delta f_i=\Delta p_i$

Approximate $\quad m\Delta\ddot{x}_i(t)+c\Delta\dot{x}_i+(k_i)_T\Delta x_i=\Delta p_i$

Unbalanced $\quad \Delta f_i-(k_i)_T\Delta x_i=\Delta R$

$$\Delta x_i^{(j)}=\frac{\Delta R^{(j)}}{(k_i)_T}$$

Newton-Raphson Iteration

$$\Delta x_i^{t}=\Delta x_i+\Delta x_i^{(1)}+\Delta x_i^{(2)}+\cdots$$

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Given m, c, k, p, x_0 and $\dot{x}_0$

Assume γ and β $(\gamma = \frac{1}{2},\ \frac{1}{6} \le \beta \le \frac{1}{4})$ Δt

1) Initialize

$$\ddot{x}_0 = \frac{p_0 - c\dot{x}_0 - (f_s)_0}{m}$$

$$a = \frac{m}{\beta \Delta t} + \frac{\gamma c}{\beta}$$

$$b = \frac{m}{2\beta} + \Delta t\left(\frac{\gamma}{2\beta} - 1\right)c$$

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$$\Delta R^{(i)} = \Delta \hat{p}_i$$

$$x_{i+1}^{(0)} = x_i$$

2) For each time step i (i = 0,1,2,…)

$$\Delta \hat{p}_i = \Delta p_i + a\dot{x}_i + b\ddot{x}_i \qquad \hat{k}_i = k_i + \frac{\gamma}{\beta \Delta t}c + \frac{1}{\beta \Delta t^2}m$$

j=1, 2, …

$$\Delta x^{(j)} = \frac{\Delta R^{(j)}}{\hat{k}_T} \qquad x_{i+1}^{(j)} = x_{i+1}^{(j-1)} + \Delta x^{(j)}$$

$$\Delta f^{(j)} = f_s^{(j)} - f_s^{(j-1)} + (\hat{k}_T - k_T)\Delta x^{(j)} \qquad \Delta R^{(j+1)} = \Delta R^{(j)} - \Delta f^{(j)}$$

$$\hat{k}_T = k_T + \frac{\gamma}{\beta \Delta t}c + \frac{1}{\beta \Delta t^2}m$$

(Newton-Raphson iteration)

$$\Delta \ddot{x}_i = \frac{1}{\beta \Delta t^2}\Delta x_i - \frac{1}{\beta \Delta t}\dot{x}_i - \frac{1}{2\beta}\ddot{x}_i$$

$$\Delta \dot{x}_i = \frac{\gamma}{\beta \Delta t}\Delta x_i - \frac{\gamma}{\beta}\dot{x}_i + \Delta t\left(1 - \frac{\gamma}{2\beta}\right)\ddot{x}_i$$