CARNOT EWS model - **Model for vertical ground heat exchanger**

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Introduction

The CARNOT EWS model is a transient model for dynamic simulation of borehole heat exchangers or so-called vertical ground heat exchangers (v-ghx) in the simulation environment Matlab/Simulink within the CARNOT Blockset.

The model is based on the equations described by [Huber97] and has been implemented using the state space model of Matlab/Simulink by [Bianchi06].

The geometric (e.g. depth and diameter of bore or type of borehole heat exchanger) and physical (e.g. ground properties) parameters as well as the operational parameters and boundary conditions are defined in a script (m-file), which is executed before simulation (initialization). Consequently, changes of the mass flow during operation/simulation could not be considered in the original model.

In the majority of cases a ground heat exchanger will be operated with constant mass flow and thus varying mass flow will rarely occur, however, simulation of discontinuous operation (on-off) is certainly required. As the original model did not allow for that the model was enhanced and on-off operation was implemented in the actual version. Furthermore, the originally used "Trichterformel" as boundary condition was changed to a boundary condition based on g-functions (Eskilson) in order to allow for multiple borehole configurations. The third modification concerns the initial conditions. Whereas in the original model the nodes of the numerical grid were initialized with a homogenous temperature field, in the actual version a temperature distribution based on the steady state solution using g-functions can be calculated.

In the following, the model is briefly described. Finally, different validation calculations have been performed, which will be presented after the description of the model.

Model description

A detailed description can be found in [Huber97] and [Huber99]. Here, the most important aspects will be summarized.

Axial and Radial heat transport in Borehole/Brine

For the calculation of the axial distribution of the brine temperature, the bore is divided in axial layers with the length dl. The flow and the return temperature are calculated separately. However, in case of double U-pipe the pipes are considered combined. The energy balance is calculated for each segment considering heat conduction in radial direction and convection (and conduction) in axial direction, according to **Figure 1**.

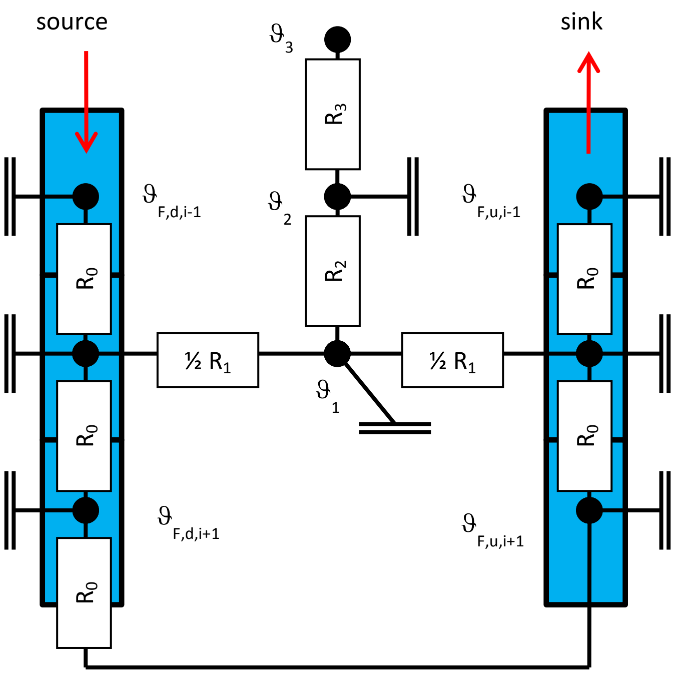


Figure 1 – Axial and radial heat transfer in the borehole, conduction and convection

The brine properties are calculated during the initialization of the model for a given average fluid temperature and are considered to be constant during the course of the simulation.

Radial heat conduction in near range ground

The numerical grid for the calculation of the radial temperature distribution is show in **Figure 2**. The layer with i = 0 represents the brine and is coupled with the brine model, explained above. The second radial layer (i = 1) represents the grout. Starting from i = 2 several ground layers can follow. The properties of the ground are considered to be homogeneous in the entire domain. The boundary condition for the brine is calculated using the average brine temperature (average between flow and return temperature).

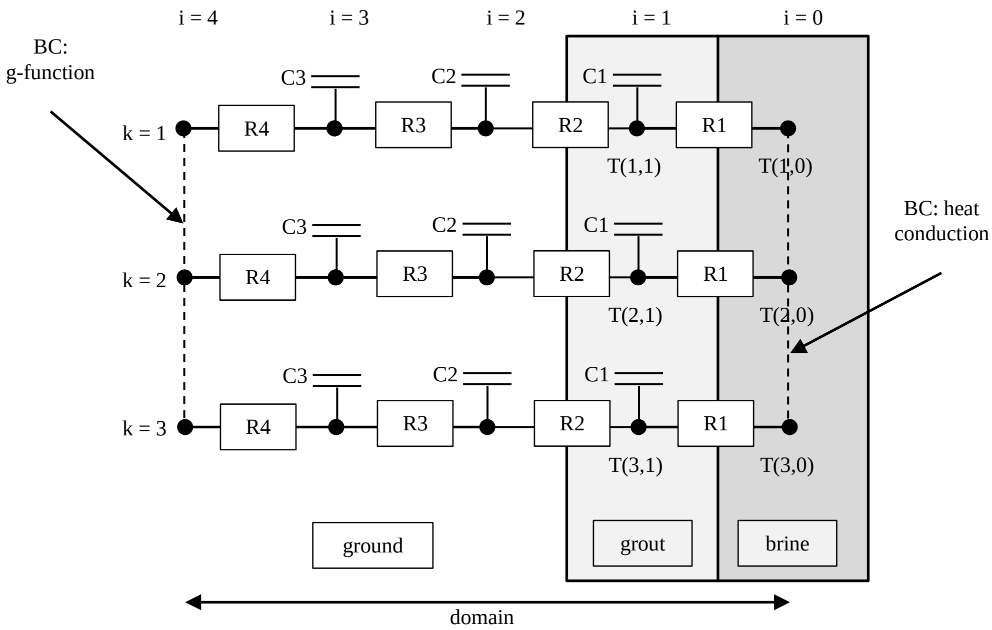


Figure 2 – Radial heat transfer according to [Bianchi06], modified

Different ground layers (ground properties depending on the depth) are not possible in the current version but may be implemented in future.

Borehole Configuration and Borehole thermal resistance

The resistance R_3 and R_4 in **Figure 1** can be derived from the Fourier equation in radial coordinates. Contrariwise, the resistances R_1 and R_2 cannot be derived exactly, as the 2-dimensional problem (see **Figure 3**) cannot be depicted 1-dimensionally without simplifications.

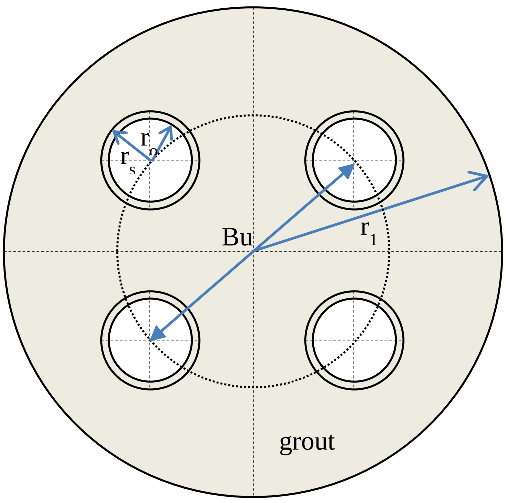


Figure 3 – Borehole configuration with internal pipe radius $r_0 = D_i/2$, borehole radius $r_1 = D_b/2$, and $rz_1 = Bu/2$, $rz_2 = ((r_2^2 + r_1^2)/2)^{0.5}$, $dl = H/n_{ax}$

The simplification of a 1-dimensional model yields the following equations for R_1 and R_2 (see [Huber])

$$R_1 = \frac{1}{4}\left(\frac{1}{2\pi\alpha r_0 dl} + \frac{1}{2\pi\lambda_{grout} dl}\ln\frac{r_1 + rz_2}{r_0}\right) \qquad 1$$

$$R_2 = \frac{1}{2\pi dl\lambda_{grout}}\ln\frac{r_1}{rz_1} + \frac{1}{\lambda_{soil}}\ln\frac{rz_2}{r_1} \qquad 2$$

The heat transfer coefficient α can be calculated using well-known Nusselt-correlations.

$$\alpha = \frac{Nu(Re, Pr)\cdot\lambda_{brine}}{D_i} \qquad 3$$

Alternatively, R_1 and R_2 can be calculated if the internal borehole resistance R_a and the borehole thermal resistance R_b are known from other programs (e.g. EED, Hellström) or from numerical simulation.

According to the definition in **Figure 1**, the resistance R1 is defined as follows:

$$R_1 = \frac{(\vartheta_f - \vartheta_1)}{\dot{Q}} \qquad 4$$

On the other hand the borehole thermal resistance is defined by the following equation:

$$R_b = \frac{dl(\vartheta_f - \vartheta_b)}{\dot{Q}} \qquad 5$$

The resistances R_1 and R_2 can thus be calculated using the following 2 equations:

$$R_1 = \frac{R_a}{4dl} \qquad 6$$

$$R_2 = \frac{\left(R_b - \frac{R_a}{4}\right)}{dl} + \frac{1}{2\pi\lambda_{soil} dl}\ln\frac{rz_2}{r_1} \qquad 7$$

Equation (7) is limited to the case that Ra < 4 * Rb.

Boundary Condition

The temperature distribution at the outer boundary of the domain is calculated using g-functions. The temperature difference ΔT resulting from the average heat flux $\dot{q}$ is updated weekly. (It is assumed that the heat extraction is constant over the course of a week.)

$$\Delta T(r,t) = \frac{\dot{q} \cdot g(t,r)}{2 \cdot \pi \cdot \lambda} \qquad 8$$

This equation applies for constant heat extraction/injection $\dot{q}$. Using the principle of superimposition time varying heat extraction can be considered. The g-function has to be given in form of a look-up-table. It can be approximated using a polynomial approach, with which published data [i.e. Eskilson] can be approximated (see [Huber]).

For configurations with more than one ground heat exchanger, the type of configuration (g-function), the distance B (for the determination of the simulation domain) and the number of parallel pipes (n_p) corresponding to the selected g-function has to be chosen.

The boundary condition is updated weekly as is recommended in [Huber].

Initial Condition

For the initial conditions can be chosen between a homogeneous temperature field (as e.g. required in the case of a thermal response test) and a steady state temperature field according to equation (9).

$$\vartheta(r,z) = \vartheta_m + \Delta T_{grad} \cdot z - \frac{g(t_{ini},r)}{2 \cdot \pi \cdot \lambda} \cdot \dot{q} \qquad 9$$

Validation

For the validation three cases have been investigated. The first validation example is measured data from a thermal response test [IEA ECES A21]. The second case is a validation against a fully discretized 3D ground heat exchanger model with simulated data of a period of heat injection from [Bauer2011]. Both cases address the short term behaviour. For the validation of the long term behaviour of the ground heat exchanger model the results are compared with calculation results of calculation methods such as EED and EWS (third case).

<u>Case 1: Thermal response test</u>

The thermal properties of the grout and the ground are summarized in Table 1. The borehole properties are listed in Table 2.

Table 1 - Thermal properties of grout and ground

	grout	**soil**
λ / [W/(m K)]	2.1	2.3
ρ / [kg/m³]	2500	2500
c / [J/(kg K)]	800	800

Table 2 - Borehole properties and parameters

type	2-U
height	H = 193 m
borehole diameter	D_b = 0.2 m
pipe	D = 40/3.7 mm
mass flow	mdot = 0.44 kg/s (water)
undisturbed ground temperature	ϑ_g = 14.7 °C

The measured and calculated development of the flow temperature (out) for a given inlet temperature (in) is shown in **Figure 4**.The simulation results are shown for the case that the resistances R_1 and R_2 are calculated based on equations (1) and (2) and for the case they are calculated based on R_a and R_b. For the latter case the agreement between simulated and measured data is very good.

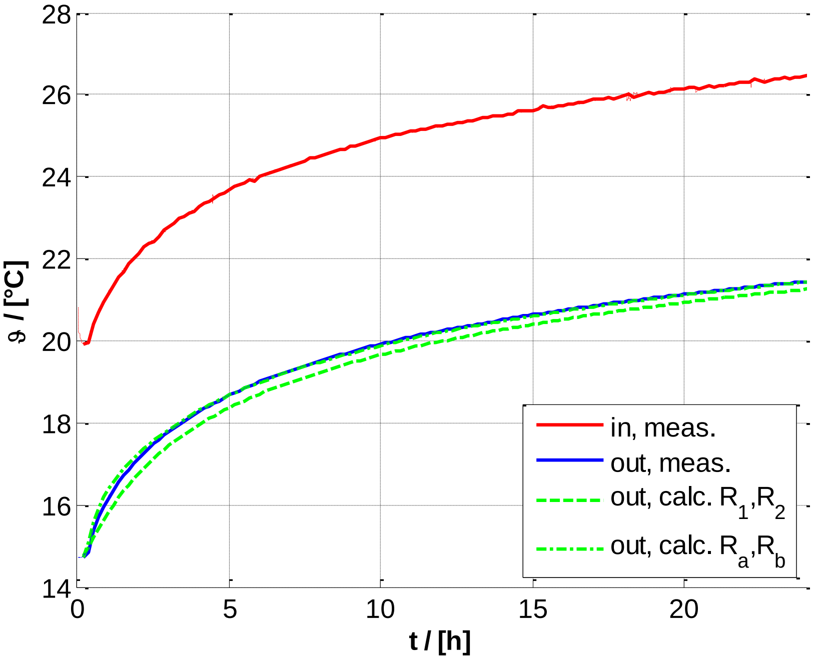


***Figure 4** – Comparison of Simulink EWS model with TRT measurement data (IEA ECES A21)*

The maximum deviation with regard to extracted heat (integrated extracted power) is below 5 % even in case of the simulation based on resistances R_1, R_2 (see Table 3).

Table 3 - Comparison of measurement and simulation based on resistances R_1, R_2

	Q (90h) / [kWh]	**Q (24 h) / [kWh]**
TRT	825.5	436.5
EWS	847.3	455.7
err / [%]	2.7	4.4

Case 2: ANSYS FEM Simulation

The thermal properties of the grout and the ground are summarized in Table 4. The borehole properties are listed in Table 5.

Table 4 - Thermal properties of grout and ground

	grout	soil
λ / [W/(m K)]	2.2	2.3
ρ / [kg/m³]	2600	1460
c / [J/(kg K)]	850	1500

Table 5: Borehole properties and parameters

type	2-U
height	H = 100 m
borehole diameter	D_b = 0.13 m
pipe	D = 32/3.0 mm
pipe distance	Bu = 0.06 m
mass flow	mdot = 0.25 kg/s (water)
undisturbed ground temperature	ϑ_g = 10.0 °C

For a charging period of 24 h with 80 °C the simulation results of the fully discretized model (out, FEM) and the results of the Simulink EWS model (out,calc) are shown in **Figure 5**.

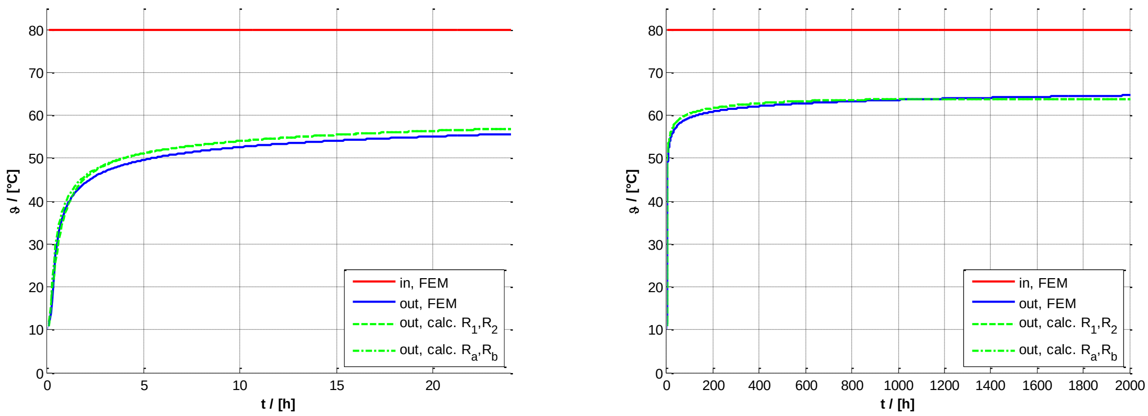


Figure 5 – Comparison of Simulink EWS model with ANSYS calculations (Bauer 2011), left first 24 h, right 2000 h

The agreement is relatively good with deviations of approx. 1 K after 24 h and a remaining deviation at the end of the simulation period, which may be a result of the simplified calculation of the resistances R_1 and R_2. The dynamic behaviour (first 3 h) agrees quite well. The error with regard to energy (integrated power) is with about 3 % in 24 h acceptable, see **Table 6**.

Table 6 - Comparison of FEM results and EWS simulation results based on resistances R_1, R_2

	Q (2160h) / [kWh]	**Q (24 h) / [kWh]**
FEM	38138.7	715.2
EWS	38085.7	690.6
err / [%]	0.14	3.4

Case 3: Long-term behaviour, comparison with EED and PHews

EED (earth energy designer) is considered as the reference tool for dimensioning of ghx. PHews is the implementation of the EWS algorithms (with some minor simplifications) in the PHPP.

For the validation of the long-term behaviour typical loads of a single family passive house is applied, see **Table 7**. Profile 1 is a typical heating (H) load and profile 2 a load typical for heating and domestic hot water (DHW). The ground properties are summarized in

Table 8 and borehole and heat exchanger parameters are listed in **Table 9**.

Table 7 – Load (heat extraction) in MWh, extraction power profile 1: P = 933 W, Q = 1444 kWh; profile 2: P = 1666 W, Q = 5623 kWh;

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Q (H)	0.43	0.21	0.09	0.03	0.00	0.00	0.00	0.00	0.00	0.03	0.22	0.43
Q (H+DHW)	1.24	0.65	0.36	0.25	0.26	0.25	0.21	0.21	0.25	0.26	0.50	1.19

Table 8 – Ground properties

	grout	**soil**
λ / [W/(m K)]	1.0	2.0
ρ / [kg/m³]	2000	2500
c / [J/(kg K)]	1000	800

Table 9 – Borehole and heat exchanger parameters

type	2U
height	50|75 | 100 m
borehole diameter	D_b = 0.18 m
pipe	D = 32/3.0 mm
pipe thermal conductivity	λ_p = 0.48 W/(m K)
pipe distance	Bu = 0.1293 m
massflow	mdot = 0.5 kg/s (water-glycol 25%)
undisturbed ground temperature	ϑ_g = 10.0 °C
Geothermal gradient	T_{Grad} = 0.025K/m (= 0.05 W/m²)

The internal borehole resistance and the borehole thermal resistance are R_a = 0.46134 (K m)/W and R_b = 0.11729 (K m)/W.

As initial condition the temperature distribution for constant heat extraction of 50 years is calculated according to equation (9) using peak extraction power. The resulting initial temperature field is shown in

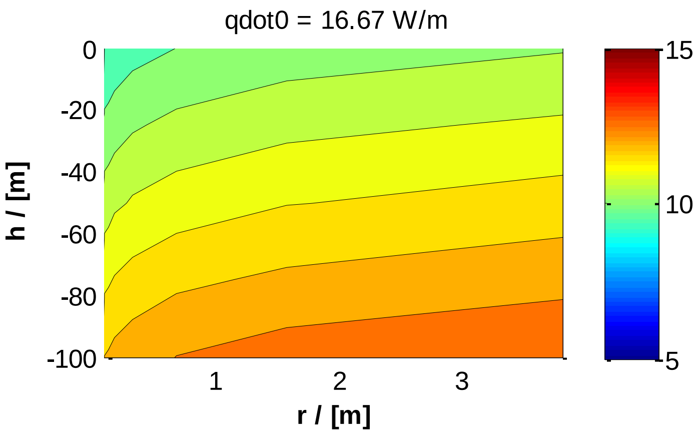


Figure 6 – Initial temperature distribution for a bore length of 100 m calculated for $\dot{q} = \dot{Q}/H$, with the peak extraction power $\dot{Q}$

The resulting temperature development for three different borehole lengths is shown in **Figure 7** for the heating profile and in **Figure 8** for the profile for heating and domestic hot water and is compared with EED and EWS simulation results.

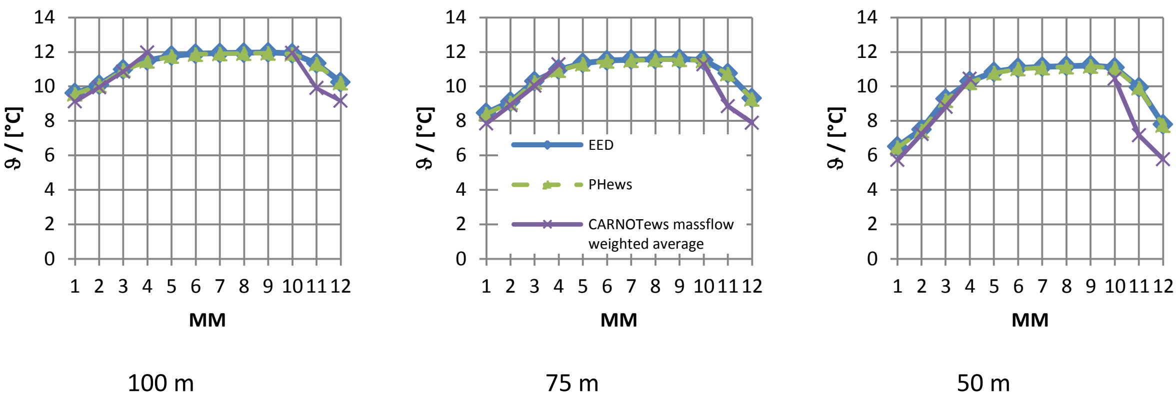
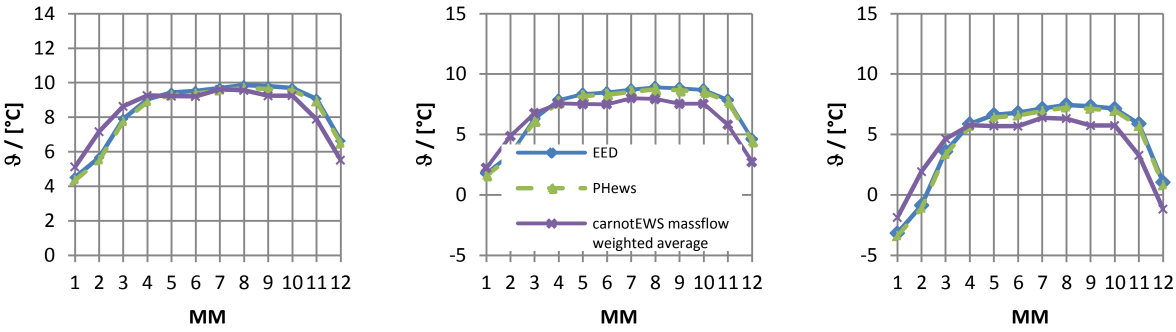


Figure 7 – Temperature development for a 100 m (left), 75 m (centre) and 50 m (right) borehole, heating profile Q(H).

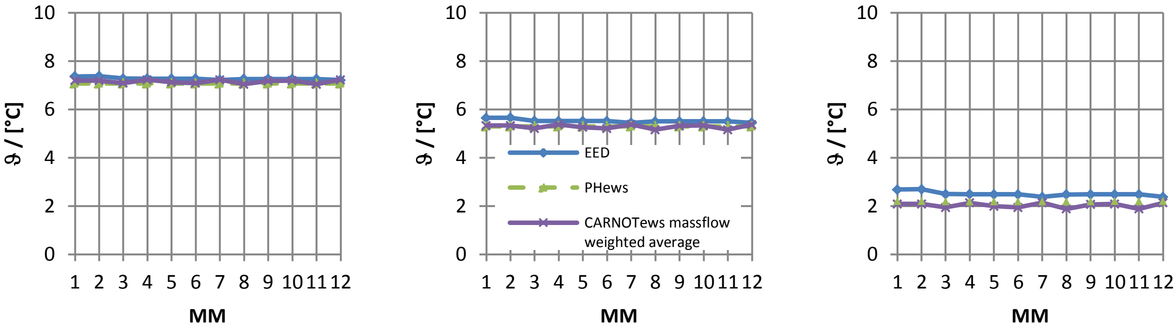


100 m 75 m 50 m

Figure 8 – Temperature development for a 100 m (left), 75 m (centre) and 50 m (right) borehole, heating profile Q(H).

The agreement between EED and PHews is very good for all cases; the results of PHews are slightly more conservative. The development of the simulated temperature (carnotEWS) is comparable with the calculation results. However, as expected, the simulated monthly massflow weighted temperatures are up 1 K to 2 K lower. The simulation yields more realistic results as actual extraction power and thus the respective borehole temperature is considered in each time step and not only the monthly energy balance as in the case of the calculation methods.

For a constant load of 12h per day with 1667 W the agreement between PHews and CARNOTews is very good as can be seen from **Figure 9**. Temperatures calculated with EED are slightly higher, in particular for higher specific loads.



100 m 75 m 50 m

Figure 9 – Temperature development for a 100 m (left), 75 m (centre) and 50 m (right) borehole, constant extraction of 12 h per day with 1667 W

Appendix: Analytical equation

$$\vartheta_{src} = \vartheta_m \cdot \Delta T_{grad} \cdot \frac{H}{2} - \left[\frac{g(t, r_b)}{2 \cdot \pi \cdot \lambda} + R_b + \frac{1}{3} \cdot \frac{1}{R_a} \cdot \frac{H^2}{\dot{m}^2 \cdot c_{p,b}{}^2} - \frac{H}{2 \cdot \dot{m} \cdot c_{p,b}} \right] \cdot \dot{q}$$

List of Parameter (SI-Units, °C)

Parameter	**Default**	**Description**
t0	0	Start time
t1	8760*3600	End time
sample_time	3600	sample time
Initial condition		
startfall	1	1: initial temperature field, 0: homogeneous temperature field
Qdot0	5000	Extraction power, calculation of temperature field
t_start	50*8760*3600	time, calculation of temperature field
operation		
mdot	0.5	mass flow
teta_in	0	inlet temperature
teta_e_m	10	average ambient temperature
borehole configuration		
sondenfall	1	1: single 2:
g-function		[4.82,5.69,6.29,. 6.57,6.6] for single bore
np	1	number of parallel bores
DimRad	10	Radial Discretization
DimAxi	10	Axial Discretization
Gitterfaktor	2.5	radius difference between two calculation volumes
Bohrdurchmesser	0.18	diameter of bore
Sondelaenge	100	length of bore
B	10	distance between two bores
Sondendurchmesser	0.032	diameter of pipe
Dicke_Sondenrohr	0.003	wall thickness of pipe
lambda_Sondenrohr	0.48	thermal conductivity of pipe
Bu	0.12	spacing (between tubes)
Ground properties		
lambdaErde	2	thermal conductivity of ground
cpErde	800	thermal capacity of ground
rhoErde	2500	density of ground
lambdaFill	2	thermal conductivity of grout
cpFill	800	thermal capacity of grout
rhoFill	2500	density of grout
TGrad	0.025	geothermal gradient
Bodenerwaermung	1	difference between ambient and surface temperature
Fluid properties		
teta_m	2	average fluid temperature
p0	1E5	pressure
fluid	5	fluid type
mix	.25	fluid mix
Boundary Condition		
Ts_Trichter	7*24*3600	time step for update of boundary condition (g-function)
Dr_Trichter	1	radius for boundary condition

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