Constrained Envelope Continuous Phase Modulation

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***Abstract*— We develop a coded modulation system with continuous phase and a small controlled envelope variation. We call the system Constrained Envelope Continuous Phase Modulation (ceCPM) and show that, there is a gain for ceCPM over CPM under the same spectrum constraint. With normalized amplitude varying in the interval $\{1 \pm 0.2\}$ the gain for ceCPM is up to 2.5 dB over CPM at symbol error probability 10^{-3}. We also show that ceCPM performs well with reduced state sequence detection (RSSD).**

I. Introduction

The continuous phase property of Continuous Phase Modulation (CPM) makes it possible to define digital modulation schemes with a narrow spectral main lobe and small spectral side lobes by using a smooth phase pulse [1]. The constant envelope property of CPM also allows the use of non-linear amplifiers, which have lower cost and significantly higher power efficiency than linear amplifiers, since they can work in a saturated mode. However, the constant envelope restricts the signal space and thus, the minimum Euclidean distance at the receiver, which has to be compensated for by larger transmitted RF power. A basic question is, how large is this penalty in the required RF power for the constant envelope property?

In this paper, we answer the question above by designing a coded modulation system with a small controlled envelope variation and comparing the performance with CPM. The proposed scheme, which we call Constrained Envelope Continuous Phase Modulation (ceCPM), has both continuous phase and continuous envelope variation, making the power spectrum to be well confined. This coded modulation system provides a gain over CPM. The gain becomes larger as the envelope is allowed to fluctuate more. We show ceCPM systems having a gain up to 2.5 dB over CPM at symbol error probability 10^{-3} while still satisfying the same power spectrum requirement.

Another important property of ceCPM is that ceCPM can be described by the same trellis as CPM, thus the number of states are not increased and as we show, the proposed system also performs well with Reduced State Sequence Detection (RSSD) [2].

The proposed modulation system challenges the power amplifier designers to design amplifiers with a local linearity close to the high efficiency working point. By using our proposed modulation scheme with such an amplifier, both high power efficiency at the transmitter and low RF power requirement at the receiver could be achieved at potentially a low amplifier cost.

In the literature there are some work devoted to finding good modulation systems with somewhat restricted envelope, e.g. multi-amplitude CPM [3] and [4, p. 199], staggered modulation formats (e.g. OQPSK see [4, p. 194]) and recently in the EDGE modulation system for evolution of GSM, see [5] for an overview of EDGE and [6] for a definition of the modulation format.

In the EDGE modulation the envelope variation is quite large and linearization techniques are necessary. In [7] a digital linearization technique for EDGE is proposed and an IQ-plot of the EDGE signal space is given. The EDGE envelope signal has a normalized envelope varying approximately in the interval $[1.56, 0.23]$, i.e. almost 17 dB dynamic range.

We illustrate ceCPM signals that improve in power efficiency more than 1.4 dB at symbol error probability 10^{-4} over CPM, while still satisfying the same spectrum requirement and with as small envelope variation as in the interval $[1.1, 0.9]$, i.e. less than 1.8 dB dynamic range. Such a small envelope variation should simplify a linearization unit.

We define ceCPM, discuss the state-description and receiver structures, and give expressions for the Euclidean distance, auto-correlation function, power spectral density and maximum/minimum amplitude. With the framework for ceCPM established, we compare the performance of ceCPM and CPM by numerically maximize the minimum Euclidean distance of the systems under a spectrum constraint defined by a spectrum mask that makes the systems to be more than twice as bandwidth efficient as GSM, and we verify the performance gain for ceCPM by simulations. Derivation of equations and much more details can be found in [8].

II. Continuous Phase Modulation

The baseband signal in CPM is defined as

$$s(t, \boldsymbol{\alpha}) = \sqrt{\frac{2E}{T}} e^{j\phi(t,\boldsymbol{\alpha})}, \qquad (1)$$

where T is the length of a symbol interval, E is the symbol energy and $\boldsymbol{\alpha}$ is the transmitted M-ary information symbol sequence with symbols taken from the set $\alpha_i \in \{\pm 1, \pm 3, \ldots, \pm(M-1)\}$. Note that the amplitude is constant for all t, thus CPM provides a constant envelope for the transmitted carrier based signal.

For symbol interval n, where $t \in [nT, (n+1)T]$, the phase function is defined as

$$\phi(t, \boldsymbol{\alpha}) = 2\pi h \sum_{i=n-L+1}^{n} \alpha_i q(t - iT) + \theta_n, \tag{2}$$

where the parameter h is called the modulation index, and the accumulated phase θ_n equals

$$\theta_n = \pi h \sum_{i=-\infty}^{n-L} \alpha_i \text{ modulo } 2\pi. \tag{3}$$

The function $q(t)$ is called the phase pulse and is a continuous function with the restriction

$$q(t) = \begin{cases} 0, & t \leq 0 \\ 1/2, & t \geq LT. \end{cases} \tag{4}$$

When restricting the modulation index to $h = k/p$, where k and p are relative prime integers, the phase state θ_n takes values from a finite discrete set. Hence, for symbol interval n, the signal (1) is determined by the state

$$s_n = [\theta_n, \alpha_{n-1}, \alpha_{n-2}, \ldots, \alpha_{n-L+1}], \tag{5}$$

and CPM can be described by a trellis with pM^{L-1} states.

III. Definition of ceCPM

In [9] it was shown that the baseband signal of binary CPM can be expressed as a linear combination of a finite number of time localized pulses, the so-called Laurent decomposition. In [10] this description was generalized for M-ary CPM, and we refer to this as the extended Laurent decomposition of CPM. Following the notation in [10], the normalized baseband signal of $M = 2^P$-ary CPM, P integer, can be described by

$$s(t, \boldsymbol{\alpha}) = \sum_{k=0}^{N-1} \sum_{n} a_{k,n} g_k(t - nT), \tag{6}$$

where as above T is the symbol time, $\boldsymbol{\alpha}$ is the M-ary information sequence, $a_{k,n}$ are the pseudo-symbols, which are non-linear functions of $\boldsymbol{\alpha}$, and $g_k(t)$ are time localized pulses called Laurent functions (LFs). The number of LFs equals $N = Q^P(2^P - 1)$, where $Q = 2^{L-1}$ and L is the length of the phase pulse in the CPM signal.

In Fig. 1 we show all LFs for 8-ary CPM with modulation index $1/8$ and GMSK phase pulse with $BT = 0.3$ truncated to 3 symbol intervals. There are 448 LFs, the 7 first of them being so-called principal LFs. As seen the principal LFs are the largest and keeping only the principal LFs will create a continuous phase signal, exhibiting envelope variations and containing 86.6% of the original signal power.

This observation has been used in the literature to derive an alternative modulator structure based on the Laurent decomposition for binary CPM with $h = 1/2$ [11] and simplified detectors for binary CPM,e.g. [12]. In e.g. [13] simplified detectors for M-ary CPM are derived based on the extended

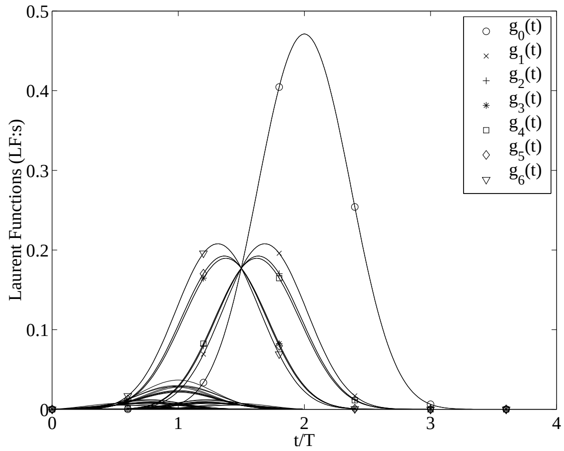

Fig. 1. All 448 LFs of the 8-ary example CPM scheme.

Laurent decomposition and synchronization parameters for CPM based on Laurent decomposition are treated in e.g. [14], see [8] for more references.

In [10] it is also discussed how to approximate a CPM signal according to the MMSE criterion by using a smaller number $N' < N$ of LFs, and an expression for the case when N' equals the number of principal LFs are derived. However, we do not try to approximate a CPM signal, but instead use the principal LFs as a starting point for defining a digital modulation technique that exhibits a small envelope variation and that utilizes this new degree of freedom to achieve an increased minimum Euclidean distance of the signal space.

In general, the ceCPM system uses $N_p = M - 1$ pulses, the number of principal LFs, with lengths being the lengths of the principal LFs [10] and the corresponding pseudo-symbol definitions i.e.

$$s(t, \boldsymbol{\alpha}) = \sqrt{\frac{2E}{T}} \frac{1}{K} \sum_{k=0}^{N_p-1} \sum_{n} a_{k,n} p_k(t - nT), \tag{7}$$

where K is a normalization constant to make the average symbol energy to be E. An expression for K is given in Section VII. Furthermore, T is the symbol time, $a_{k,n}$ are the pseudo-symbols as defined in [10] and $p_k(t)$ are time localized pulses with the same length as the principal LFs. For convenience, let us call these pulses ceLFs. As a starting point, the ceLFs can be chosen to be the principal LFs, but that will lead to a signal with less confined spectrum. We show that there are better choices.

IV. State-Description

As observed in [15], [10], for non-binary CPM $\boldsymbol{\alpha}$ can be viewed as parallel binary bit-streams. Assuming M-ary symbols with $P = \log_2(M)$ being an integer, the normalized baseband CPM signal can be expressed as

$$s(t, \boldsymbol{\alpha}) = \prod_{l=0}^{P-1} \exp\left(j2\pi h^{(l)} \sum_{i} \nu_i^{(l)} q(t - iT)\right), \tag{8}$$

where $h^{(l)} = 2^l h$ and $\nu_i^{(l)} \in \{\pm 1\}$ are the binary symbols in bit stream $\boldsymbol{\nu}^{(l)}$. The M-ary symbols α_n are related to the binary symbols $\nu_n^{(l)}$ as

$$\alpha_n = \sum_{l=0}^{P-1} 2^l \nu_n^{(l)}. \tag{9}$$

A general expression for the pseudo-symbols $a_{k,n}$ can be found in [10], but this expression can be simplified for the principal pseudo-symbols [8]

$$a_{k,n} = \prod_{l=0}^{P-1} (e^{j\pi h^{(l)} \nu_n^{(l)}})^{1-e_{0,l}^{(k)}} \sigma_n,\ 0 \le k \le 2^P - 2, \tag{10}$$

where the shift parameters $e_{0,l}^{(k)} \in \{0, 1\}$ for bit level l are derived from k via the radix-2 representation of k given by

$$k = \sum_{l=0}^{P-1} 2^l e_{0,l}^{(k)}, \tag{11}$$

and the state variable σ_n relates to the accumulated phase in CPM as

$$\sigma_{n-L+1} = \exp\left(j\pi h \sum_{m=-\infty}^{n-L} \alpha_m \right) = e^{j\theta_n}. \tag{12}$$

Thus, there is a one-to-one relation for the state-space of ceCPM to the state space of CPM (5).

V. Receivers

By observing that the state spaces for CPM and ceCPM can be chosen to be the same, the optimum receiver structure for CPM in AWGN can be reused. Hence, the optimum receiver for ceCPM in AWGN performs Maximum Likelihood Sequence Detection (MLSD) and can be implemented by the Viterbi algorithm. The only difference is that the metric is the Euclidean distance instead of the correlation metric, since the energy per symbol interval is not constant for ceCPM.

An important consequence for a practical system of the one-to-one relation between the state spaces for CPM and ceCPM is that it is straight-forward to use the Reduced State Sequence Detection (RSSD) algorithm in [2], [16] also for ceCPM. Only the correlation metric has to be replaced by the Euclidean distance metric.

VI. Euclidean Distance

An error event for ceCPM can be defined in the same way as for CPM [1]. For CPM the normalized Euclidean distance can be calculated by a function based on $d^2(\boldsymbol{\gamma})$, where $\boldsymbol{\gamma} = \boldsymbol{\alpha}_1 - \boldsymbol{\alpha}_2$. For ceCPM this is not the case, and we refer to an error event as being of *type* $\boldsymbol{\gamma}$, but the Euclidean distance depends on the actual pair $\{\boldsymbol{\alpha}_1, \boldsymbol{\alpha}_2\}$ causing the difference sequence $\boldsymbol{\gamma} = \boldsymbol{\alpha}_1 - \boldsymbol{\alpha}_2$.

It is convenient to define the minimum Euclidean distance for an error event type $\boldsymbol{\gamma}$ as

$$d_{\boldsymbol{\gamma},\min} = \min_{\{\boldsymbol{\alpha}_1,\boldsymbol{\alpha}_2 : \boldsymbol{\alpha}_1 - \boldsymbol{\alpha}_2 = \boldsymbol{\gamma}\}} d_{\boldsymbol{\gamma}}^2(\boldsymbol{\alpha}_1, \boldsymbol{\alpha}_2). \tag{13}$$

The Euclidean distance for a finite length error event (14) can be calculated by finite integration and summation limits [8]

$$d^2(\boldsymbol{\alpha}_1, \boldsymbol{\alpha}_2) = \frac{1}{4E_b} \parallel s(t, \boldsymbol{\alpha}_1) - s(t, \boldsymbol{\alpha}_2) \parallel^2 = \frac{\log_2(M)}{2K^2 T} \times \int_0^{(L_{\boldsymbol{\gamma}}+L-1)} \left| \sum_{k=0}^{N_p-1} \sum_{n=0}^{L_{\boldsymbol{\gamma}}-1} (a_{k,n}^{(1)} - a_{k,n}^{(2)}) p_k(t - nT) \right|^2 dt, \tag{14}$$

where we assume the error event starts at symbol interval $n = 0$, $L_{\boldsymbol{\gamma}}$ is the number of symbol intervals that is spanned by the first and the last non-zero difference symbol in $\boldsymbol{\gamma}$, $E_b = E/\log_2(M)$ is the energy per bit and we denote the pseudo-symbols derived from $\boldsymbol{\alpha}_i$ with $\{a_{k,n}^{(i)}\}$, $i \in \{1, 2\}$.

For ceCPM all distinct pairs $\{\boldsymbol{\alpha}_1, \boldsymbol{\alpha}_2\}$ have to be considered in order to find the minimum Euclidean distance, except for some symmetries, see [8]. We keep the difference sequence terminology because we consider small envelope variations and derive ceCPM from CPM. Hence, we expect that members of the same set of signal pairs $\{\boldsymbol{\alpha}_1, \boldsymbol{\alpha}_2\}$ as in CPM will cause Euclidean distances close to the minimum Euclidean distance.

We focus on spectrally efficient systems, which have many states. Thus, we consider just a few error event types that are the limiting error events in CPM and optimize the ceCPM scheme with respect to these. A simulation is then necessary to assess the performance.

VII. Auto-Correlation Function

The ceCPM signals form a cyclo-stationary process and in [8] we derive the average auto-correlation function for the normalized ceCPM signal (i.e. assuming $2E/T = 1$ in (7)). Using the ceil and the floor functions, the expression for the average auto-correlation function becomes

$$\bar{R}(\tau) = \frac{1}{2K^2 T} \sum_{k=0}^{N_p-1} \sum_{i=0}^{N_p-1} \sum_{q=\lfloor \tau/T \rfloor - L}^{\lceil \tau/T \rceil + L} A_{k,i}(q) \times \int_0^{(L+1)T} p_k(t) p_i(t + \tau - qT)\, dt, \tag{15}$$

where we use the common definition $\lceil x \rceil = \lfloor x \rfloor$ when x is an integer number.

The normalization assumption implies $\bar{R}(0) = 1/2$ and this gives us an expression for the normalization constant K in (7)

$$K^2 = \sum_{k=0}^{N_p-1} \sum_{i=0}^{N_p-1} \sum_{q=-L}^{L} A_{k,i}(q) \frac{1}{T} \int_0^{(L+1)T} p_k(t) p_i(t - qT)\, dt. \tag{16}$$

Here, $A_{k,i}(q)$ is the correlation of the pseudo-symbols. Since we assume that M is a power of 2, the individual bit streams $\boldsymbol{\nu}^{(l)}$ are independent and we show in [8] that for the principal pseudo-symbols we get

$$A_{k,i}(q) = \prod_{l=0}^{P-1} (\cos \pi h^{(l)})^{|q + e_{0,l}^{(k)} - e_{0,l}^{(i)}|}, \tag{17}$$

when all symbols α_n are i.i.d. with uniform distribution.

VIII. Power Spectral Density

In [8] we show that the power spectral density of ceCPM can be calculated from the auto-correlation function (15) by a finite integral expression as for CPM [4, p. 207] or [1, p. 149]

$$\Phi(f) = 2\mathrm{Re}\left\{ \int_0^{LT} \bar{R}(\tau) e^{-j2\pi f\tau}\, d\tau + \frac{1}{1-\psi(jh)e^{-j2\pi fT}} \int_{LT}^{(L+1)T} \bar{R}(\tau)e^{-j2\pi f\tau}\, d\tau \right\}, \quad (18)$$

where the characteristic function $\psi(jh) = \mathrm{E}[e^{j\pi h\alpha_n}]$ of the i.i.d. symbols α_n with uniform distribution becomes

$$\psi(jh) = \frac{1}{M}\frac{\sin(M\pi h)}{\sin(\pi h)}. \quad (19)$$

We have not considered the case when the modulation index h is an integer number. Most probably such a modulation index will create spectral impulses as for CPM.

IX. Maximum/Minimum Amplitude

In the design of a ceCPM system we want to keep track of the maximum and minimum amplitude of the signal and assume that the amplifier is perfectly linear in that (small) envelope range. Unfortunately there is not a unique sequence that gives the minimum amplitude independent of the choice of ceLFs, but in [8] we show that the amplitude is independent of the accumulated phase state and we discuss some symmetry in order to minimize the number of sequences to consider.

For later convenience, we define the normalized maximum and minimum amplitude of ceCPM as

$$A_{\max} = \max_{\{\alpha_n\}_{n=-L+1}^{0}} \sqrt{\frac{|s(t,\boldsymbol{\alpha})|^2}{2E/T}} \quad (20)$$

and

$$A_{\min} = \min_{\{\alpha_n\}_{n=-L+1}^{0}} \sqrt{\frac{|s(t,\boldsymbol{\alpha})|^2}{2E/T}}. \quad (21)$$

X. Results and Discussion

We present performance results for CPM and ceCPM schemes that satisfy the spectrum mask shown in Fig. 2 making the systems to be twice as bandwidth efficient as the GSM system. All schemes are 8-ary with $L = 3$ and the CPM scheme uses $h = 1/8$ and GMSK phase pulse [8] with $BT = 0.3$.

The ceLFs are optimized with spectrum mask and maximum/minimum amplitude as constraints and the minimum Euclidean distance as the object function. We use a similar technique as in [17] to perform the numerical optimization, see [8] for details.

In Table I we show the results for different amplitude restrictions $\{A_{\max}, A_{\min}\}$ as defined in (20) and (21). There is a large gain in minimum Euclidean distance over CPM and the gain becomes larger with increased amplitude variation.

In Fig. 3 we show the numerically optimized ceLFs and an IQ-plot of the resulting ceCPM signal for the ceCPM[0.1] scheme in Table I. The envelope is seen to vary only within the small dynamic range $\{1.1, 0.9\}$.

TABLE I
Euclidean distances for CPM and ceCPM schemes.

Scheme	Amplitude		$d_{\boldsymbol{\gamma},\min}$	
	$A_{\max}$	$A_{\min}$	$\boldsymbol{\gamma} = [2,-2]$	$\boldsymbol{\gamma} = [2,-4,2]$
CPM	1	1	0.426	0.422
ceCPM[0.1]	1.100	0.900	0.519	0.629
ceCPM[0.15]	1.150	0.850	0.596	0.636
ceCPM[0.2]	1.200	0.800	0.690	0.714

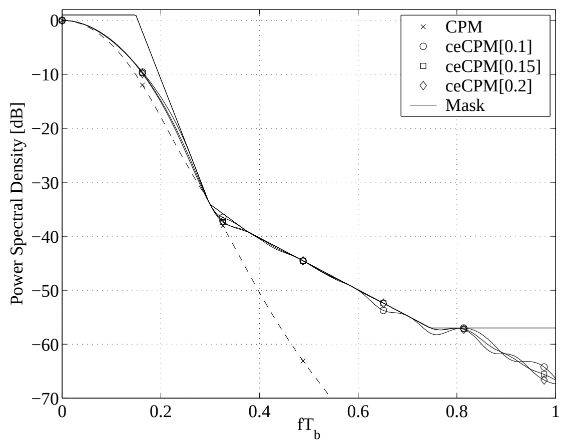

Fig. 2. Spectrum mask and power spectral densities of the schemes in Table I.

The optimal MLSD receiver for all the presented schemes in Table I have $pM^{L-1} = 512$ states. In Fig. 4 we show the performance of receivers in AWGN using MLSD and RSSD based on the state fusing vector $RS = [2, 8, 1]$ as defined in [16]. The receiver has only 16 states and the performance is very close to the MLSD receiver. The performance gain over CPM is up to 2.5 dB at symbol error probability 10^{-3}. This gain is larger than the asymptotic gain predicted by the minimum Euclidean distance, since ceCPM has a lower weight spectrum than CPM for the small Euclidean distances. The scheme ceCPM[0.2] has somewhat larger penalty with RSSD at low SNR, but for larger SNR RSSD performs close to MLSD. The other ceCPM schemes perform almost equally well with RSSD as with MLSD. Note that we have not optimized the state reduction vector, and we have not assessed the performance of RSSD for ceCPM with adjacent-channel interference and sub-optimum receiver filters as in [16].

We estimate the performance of ceCPM for other envelope restrictions by plotting $d^2_{\min}$ as a function of $(\Delta A)^2$ in Fig. 5, i.e. $\{A_{\max}, A_{\min}\} = \{1 \pm \Delta A\}$. In this figure we also include the result of CPM using an optimized phase pulse with $h = 1/8$ and $L = 3$ as given in [17] (i.e. $(\Delta A)^2 = 0$ and $d^2_{\min} = 0.441$). As seen, the increase in minimum Euclidean distance is almost linear in the envelope variation energy.

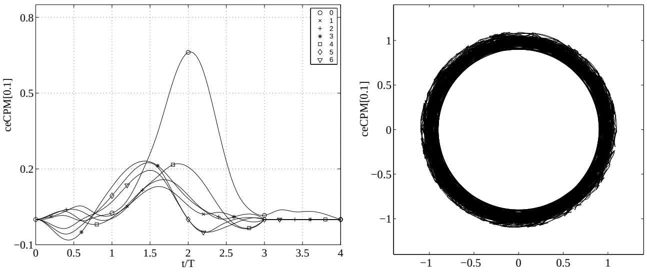

Fig. 3. Normalized ceLFs (left) and IQ-plot (right) of the ceCPM[0.1] scheme.

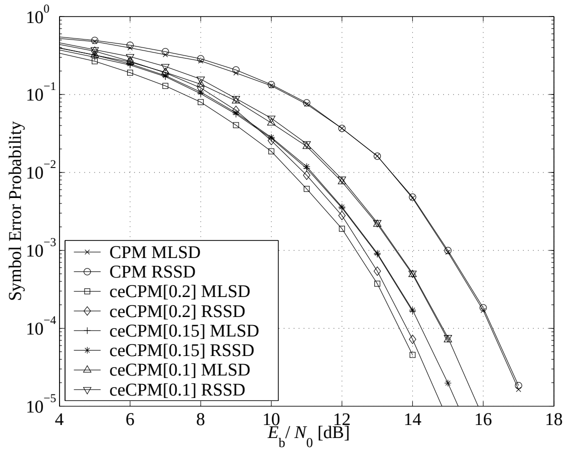

Fig. 4. Simulated symbol error probability for the reference CPM scheme and the ceCPM schemes in Table I using MLSD and RSSD with state fusing vector $RS = [2, 8, 1]$ in AWGN with one-sided power spectral density N_0.

XI. Conclusion

In this paper, we presented a digital modulation system with continuous phase and controlled envelope variation. The modulation system is referred to as Constrained Envelope Continuous Phase Modulation (ceCPM) and is derived from the extended Laurent decomposition of CPM. We showed the state description, receiver structures and expressions for Euclidean distance, auto-correlation function, power spectral density and maximum/minimum amplitude. The expressions for power spectral density and auto-correlation function could be useful also in the analysis of synchronization algorithms and simplified receivers for CPM that use Laurent decomposition.

We showed, in a spectrally efficient scenario, that there is a significant gain for ceCPM over CPM under the same spectrum constraint, and we demonstrated that ceCPM performs well with Reduced State Sequence Detection.

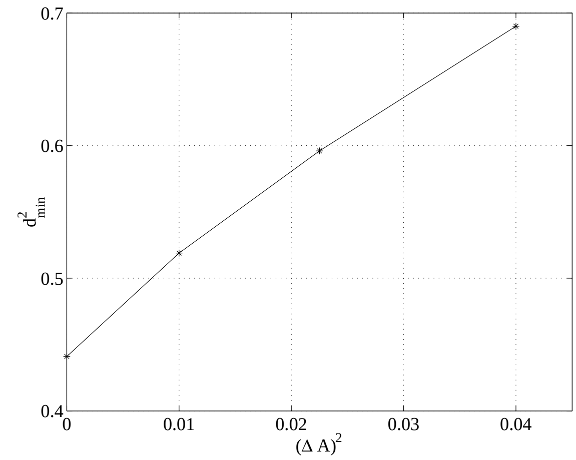

Fig. 5. Minimum Euclidean distance as a function of envelope variation for 8-ary ceCPM.

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