The Costs of Financial Distress across Industries

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Abstract

I estimate the market's opinion of ex-ante costs of financial distress (CFD) from a structurally motivated model of the industry, using a panel dataset of monthly market values of debt and equity for 269 firms in 23 industries between 1994 and 2004. CFD are identified from market values and betas of a company's debt and equity. The market expects costs of financial distress of 5% of firm value for observed leverage ratios. In bankruptcy, distress costs can rise as high as 31%. Across industries, CFD are driven primarily by the potential for debt overhang problems and distressed asset fire-sales. There is considerable empirical support for the hypothesis that firms choose a leverage ratio based on the trade-off between tax benefits and CFD. The results do not confirm the under-leverage puzzle for firms with publicly traded debt.

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Costs of financial distress (CFD) are an important component of the Trade-Off theory of optimal capital structure. Based on the Modigliani-Miller (1958) result, this paper derives a new relationship between a firm's share price, its systematic risk (beta), and its cost of financial distress. This relation separates financial costs from economic costs of distress, and forms the basis for a structural empirical model that separately estimates these costs. The separation of economic and financial distress is important because only the costs of *financial* distress matter for optimal capital structure.

I estimate the model on a sample of 269 U.S. companies in 23 industries, using a novel Markov Chain Monte Carlo (MCMC) procedure. Within the sample, ex-ante expected CFD are 5% of firm value on average, and vary between 0 and 16% across industries. At bankruptcy, CFD are as high as 31% of firm value. Much of the variation in distress costs across industries is determined by two drivers. First, industries with large growth opportunities (measured as high research and development expenses and market-to-book-ratios) tend to have high potential CFD, consistent with the debt overhang problem (Myers, 1977). The second driver of distress costs is the asset fire-sale discount (Shleifer and Vishny, 1992), which I measure as the proportion of intangible assets. Human capital, product uniqueness (Titman, 1984) and reliance on bank debt also have a marginal economic impact on CFD. I do not find that the ease of refinancing is a major determinant of CFD.

Industries with higher potential costs of financial distress adopt lower levels of leverage. Generally, the model predicts optimal capital structures that are close to observed capital structures, suggesting that the magnitude of the under-leverage puzzle (Graham, 2000) is sensitive to the measurement of costs of financial distress. Measuring CFD carefully, I find that the puzzle appears less severe for companies with publicly traded debt.

Empirical studies of CFD face a fundamental problem of separating financial costs from economic costs of distress. This problem arises because financial distress is often caused by economic distress, and it is difficult to separate an observed drop in firm value into the value lost due to a deteriorating business (economic distress) and the value lost due to the increase in the chance of default induced by the firm's debt (financial distress).

I solve this identification problem by exploring a relationship between CFD and systematic risks (betas) of debt and equity derived from the Modigliani-Miller result. Identification comes from the insight that the magnitude of the CFD affects how a change in leverage translates into changes in the betas of debt and equity. For example, for a firm with large CFD, a small increase in leverage leads to a large drop in the value of equity. Consequently, the equity beta is larger than the standard MM relationship (without costs of financial distress) predicts. Assuming a constant asset beta across a cross-section of firms within each industry, I recover implied CFD from differences in leverage and differences in systematic risks of their debt and equity.

The identification relies on a number of assumptions. First, I assume that within industries, firms have the same asset betas. Simulations (in appendix C) show that the results are robust to reasonable violations of this assumption. Second, firms in an industry are assumed to have the same costs of financial distress at the same level of leverage. Both assumptions are likely to hold when firms within an industry are similar in terms of the types of assets in place, growth opportunities and production technology. Although I do not empirically pursue other specifications of CFD here, the identification argument applies more generally to situations where CFD are a function of the firm's observable characteristics, such as credit ratings and market-to-book ratios, and can also depend on the value and risk of the unlevered assets. However, when CFD are a function of unobserved characteristics, an endogeneity problem raises additional complications.

The analysis focuses on measuring the costs of financial distress. Firms also realize a benefit of the tax shield arising from the deductibility of interest payments. In principle, the model identifies the effect of costs of financial distress net of the value of the tax shield, but two simple assumptions about the tax benefits suffice to calculate upper and lower bounds on CFD. For the purpose of comparing optimal and observed capital structures it is not necessary to separate tax benefits and CFD, because a firm's optimal capital structure only depends on the net effect.

Few papers in the empirical literature attempt to measure the magnitude of costs of financial distress. The seminal study by Altman (1984) finds sizeable costs of distress but does not break them down into the financial and economic components. Summers and Cutler (1988) exploit a lawsuit between Texaco and Pennzoil to separate these costs and conclude that ex-ante CFD are around 9% of Texaco's value. Andrade and Kaplan (1998) investigate a sample of 31 companies that became distressed after undergoing leveraged buyouts. They find ex-post costs of distress between 10 and 23% of firm value and conclude that the costs are modest, but acknowledge that low CFD may be the reason these firms were highly levered initially. The methodology developed in this paper does not rely on a specific event, such as a lawsuit or LBO. It applies to any sample of firms, and the analysis complements prior studies by employing a substantially larger dataset. Finally, Almeida and Philippon (2006) use the ex-post CFD of Andrade and Kaplan and calculate the ex-ante costs of financial distress using risk-neutral probabilities of default in a multi-period setting. Consistent with the results in this paper, they find CFD of up to 13% of firm value for investment grade firms.

The data consists of a panel of monthly data on 269 publicly traded companies in 23 industries, between 1994 and 2004. Using a novel MCMC procedure (see Robert and Casella, 1999, and Carter and Kohn, 1994), I estimate ex-ante CFD that include the direct and indirect costs of financial distress that are realized both before and after default. This is more general than the established way of estimating ex-ante CFD as the product of the probability of default and a "loss-given-default" (e.g. Leland, 1994, and Almeida and Philippon, 2006), which implies that there is no loss absent default. It is important to take into account the costs of financial distress that occur before default because these losses can be substantial even if the company never files for bankruptcy.[1] The estimation accounts for the uncertainty in estimating betas of infrequently traded corporate bonds, but faces a missing variables problem since the market values of bank debt and capitalized leases are unobserved. To assess the severity of this problem, I estimate the model under two alternative sets of assumptions, providing upper and lower bounds on the unobserved debt valuations, and find that estimated CFD are robust across these specifications.

[1]Many distressed companies restructure outside of court (Gilson, 1997). Andrade and Kaplan (1998) find that a substantial portion of the costs are suffered *before* a Chapter 11 filing.

The paper is organized as follows: the next section explores the relation between costs of financial distress and the market values and betas of corporate debt and equity, and how this relation can be inverted to identify the costs of financial distress. Section II explains the estimation methodology that applies the model to the data. The data is presented in section III. I discuss the results in section IV, and section V concludes.

I Identification of the Costs of Financial Distress

In this section I first generalize the Modigliani-Miller (1958) relations to show how the market discounts all CFD into the market prices and betas of a company's securities. I then present the identification assumptions that allow for the estimation of expected CFD from the market prices and betas of corporate debt and equity.

A Modigliani-Miller with Costs of Financial Distress

Modigliani and Miller consider the firm as a portfolio of all outstanding claims on the company. The total market value of the company at time t, V_t^L, is the sum of the market values of the individual claims:

$$V_t^L = D_t + E_t \tag{1}$$

D_t and E_t are the market value of corporate debt and equity, respectively, at time t.[2]

A different way of decomposing the same company is as:

$$V_t^L = V_t^U + B_t \tag{2}$$

where V_t^U is the market value of the *unlevered* firm. It is equal to the value of the company at time t if all its debt were repurchased by its shareholders. Interest tax shields and costs of financial distress cause V_t^U to be different from V_t^L, and therefore V_t^U is never directly observed (unless the firm truly has no debt). The difference between V_t^L and V_t^U is a fictitious security, B_t, which is defined as the expected present value at time t of the benefits of debt financing, BDF_t, minus the present value of lost future cash flows due to past financing decisions, CFD_t:

$$B_t = BDF_t - CFD_t \tag{3}$$

The benefits of debt financing include interest tax shields and decreases in agency costs due to the presence of debt in the firm's capital structure, such as the reduction in free cash flows that managers can spend on perks or unproductive pet projects (Jensen, 1986). The market discounts all expected future CFD, so CFD_t includes the direct and indirect CFD that are realized both before and after

[2]The debt and equity claims can be decomposed further into corporate bonds, bank debt and capitalized leases, and common and preferred equity, but it is not necessary to do so for the purpose of this paper.

default, and is on an ex-ante basis.[3] A positive B_t means that the benefits of debt financing outweigh the costs of financial distress, and a company is worth more with debt in its capital structure than it is worth without debt.

The company also has systematic risk, β_t^L, proportional to the (conditional) covariance of returns to the firm with some risk factor.[4] The decomposition of the firm as a portfolio of debt and equity securities yields:

$$\beta_t^L = \frac{D_t}{V_t^L}\beta_t^D + \frac{E_t}{V_t^L}\beta_t^E \tag{4}$$

The firm's systematic risk is a weighted average of the debt and equity betas, β_t^D and β_t^E. Since these betas can be estimated from observed data, β_t^L can in principle be calculated.

Using the decomposition of the company as the value of unlevered assets and tax benefits net of CFD, the beta of the levered firm can equivalently be written as:

$$\beta_t^L = \frac{V_t^U}{V_t^L}\beta_t^U + \frac{B_t}{V_t^L}\beta_t^B \tag{5}$$

By definition, the systematic risk of the unlevered assets, β_t^U, is not affected by the capital structure of the firm. The effect of leverage on the beta of the levered firm, β_t^L, is driven entirely by the net benefit of debt financing, B_t, and its systematic risk, β_t^B. We can decompose β_t^B further as:

$$\beta_t^B = \frac{BDF_t}{B_t}\beta_t^{BDF} - \frac{CFD_t}{B_t}\beta_t^{CFD} \tag{6}$$

When tax shields dominate, $B_t > 0$ and β_t^L is lower than the beta of the unlevered firm, β_t^U, because the tax shield is less risky than the firm's assets: $\beta_t^{BDF} < \beta_t^U$. This is analogous to investing in a portfolio of two securities with positive betas, where each security has a positive weight. When CFD become large, $\beta_t^L > \beta_t^U$ because the weight of the portfolio invested in the unlevered assets becomes larger than 1 ($V_t^U/V_t^L > 1$ when $B_t < 0$). In addition, costs of financial distress amplify the economic shocks to the firm; bad states become worse because in addition to a bad economic shock, the costs of financial distress increase, causing the firm to lose even more value (and vice versa for good shocks). Therefore, β_t^{CFD} has the opposite sign of β_t^U, which implies that β_t^B has the opposite sign of β_t^U when $B_t < 0$. The effect of CFD on β_t^L is therefore equivalent to shorting a negative beta security to invest in a positive beta security. Note that since V_t^U and B_t are unobserved, their betas are unobserved as well.

[3] Examples of CFD are the impaired ability to do business due to customers' concerns for parts, service and warranty interruptions or cancelations if the firm files for bankruptcy (Titman and Opler, 1994), investment distortions due to debt overhang (Myers, 1977) and asset substitution (Jensen and Meckling, 1976), distressed asset fire-sales (Shleifer and Vishny, 1992), employees leaving the firm or spending their time looking for another job, and management spending much of its time talking to creditors and investment bankers about reorganization and refinancing plans instead of running the business.

[4] At this point it does not matter what the risk factor is, or how many risk factors there are. In the empirical implementation I use the beta with the market portfolio.

By the Modigliani-Miller arbitrage argument, the market values and betas of the two portfolio decompositions of the firm have to be equal:

$$V_t^U + B_t = D_t + E_t \quad (7)$$

$$\frac{V_t^U}{V_t^L}\beta_t^U + \frac{B_t}{V_t^L}\beta_t^B = \frac{D_t}{V_t^L}\beta_t^D + \frac{E_t}{V_t^L}\beta_t^E \quad (8)$$

The first equation states that the market values of the two portfolios, expressed in equations (1) and (2), have to be the same. Equation (8) is derived by equating (4) and (5), and captures the mechanical relation between the asset beta (β_t^U) and the betas of the net value of debt financing, corporate debt and equity (for a proof, see appendix A).

To illustrate the effect of tax benefits and costs of financial distress on the value and beta of the levered firm, I will first consider two traditional cases: the Modigliani-Miller (1958) case with no taxes and no CFD, and the case of constant marginal tax rates and no CFD. Then I consider the same two cases but include costs of financial distress.

In the traditional Modigliani-Miller (1958) case with no tax benefits and no costs of financial distress, $B_t = 0$. Equations (7)-(8) reduce to the well-known formulas:

$$V_t^U = D_t + E_t \quad (9)$$

$$\beta_t^U = \frac{D_t}{V_t^U}\beta_t^D + \frac{E_t}{V_t^U}\beta_t^E \quad (10)$$

By equations (1) and (4), the right side of (9) and (10) are the value and the beta of the levered firm, V_t^L and β_t^L, respectively. Both V_t^L and β_t^L are unaffected by the leverage ratio $L_t \equiv D_t/V_t^L$. The top-left graph in figure 1 illustrates how the betas of debt and equity vary with leverage.

In the presence of a constant marginal tax rate, τ, but no costs of financial distress, Bierman and Oldfield (1979) show that the present value of the tax shield is $BDF_t = \tau D_t$. This implies that $B_t = \tau D_t$, since $CFD_t = 0$ by assumption. Equation (7) then becomes $V_t^L = V_t^U + \tau D_t$, i.e. the value of the levered firm equals the value of the unlevered firm plus the present value of the interest tax shield. From the expression for B_t it follows that the return to B is equal to the return to debt, so that $\beta_t^B = \beta_t^D$. Plugging this into equation (8) yields:

$$\frac{V_t^U}{V_t^L}\beta_t^U = (1-\tau)\frac{D_t}{V_t^L}\beta_t^D + \frac{E_t}{V_t^L}\beta_t^E \quad (11)$$

The top-right graph in figure 1 shows how the beta of the levered firm decreases as financial leverage increases. Assuming in addition that β_t^D equals zero results in the standard textbook formula $\beta_t^E = \left(1 + \frac{(1-\tau)D_t}{E_t}\right)\beta_t^U$ (see for example Ross et al., 1996, p.469).

Whereas tax benefits increase the value of the levered firm, costs of financial distress have the opposite effect. Without tax benefits but in the presence of costs of financial distress, the bottom-left plot in figure 1 illustrates that the levered firm's beta, β_t^L, increases with leverage. This relation implies that it is optimal for the firm to have no debt in its optimal capital structure.

With both tax benefits and costs of financial distress, the company's market value becomes a hump-shaped function of leverage, and thus the levered firm's beta becomes a U-shaped function of leverage, as illustrated in the bottom-right graph of figure 1. This is consistent with the Trade-Off theory of optimal capital structure, in which firms choose the leverage ratio that maximizes firm value. The This is a result of the trade-off between tax benefits and costs of financial distress: whereas tax benefits reduce the firm's beta when financial leverage is relatively low, costs of financial distress counter this effect as leverage increases.

As these examples show, the way the riskiness of the firm, as measured by its beta, changes with leverage is highly dependent on the existence and magnitude of tax benefits and costs of financial distress. In the next section I exploit this relation to identify the benefits and costs of financial leverage that matches the variation in levered firm betas within an industry.

B Identification

The existing literature takes the value equation (7) and treats identification of B_t as a missing variables problem. Even if the value of the levered firm, $D_t + E_t$, is observed, both B_t and V_t^U are unobserved. It is therefore not possible to recover B_t from equation (7) alone. Consider the approach in econometric terms by rewriting equation (7) to have B_t on the left-hand side:

$$B_t = (D_t + E_t) - V_t^U \tag{12}$$

Take first differences:

$$\Delta B_t = \Delta(D_t + E_t) - \Delta V_t^U \tag{13}$$

In this setup, the ΔV_t^U term is a missing variable. One can only observe the change in the value of the *levered* firm, $\Delta(D_t + E_t)$, whereas the *unlevered* firm is not traded. In other words, it is not possible to separate an observed drop in the value of the levered firm into a drop in V_t^U (economic distress) and an decrease in B_t (financial distress). Treating ΔV_t^U as an error term leads to an endogeneity problem because it is correlated with the change in levered firm value. To resolve this issue, previous studies rely on natural experiments that exogenously change financial leverage, while leaving the unlevered firm value unchanged ($\Delta V_t^U = 0$). Such experiments function like instruments that are correlated with $\Delta(D_t+E_t)$ but not with the error term ΔV_t^U. Examples of such experiments are lawsuits (Summers and Cutler, 1988) and leveraged buy-outs (Andrade and Kaplan, 1998).

The natural experiment approach has the advantage of being transparent and requiring relatively few assumptions. However, it has proven difficult to find suitable experiments that generate large samples. The largest sample that has been used up to date is by Andrade and Kaplan (1998) and comprises 31 firms, of which 13 did not suffer an adverse economic shock ($\Delta V_t^U = 0$). Moreover, the nature of most experiments introduces a selection bias into the sample, making it difficult to judge the generality of the results. The quality of the instrument is an issue, especially since changes in values are measured over a time frame of years. The question is whether ΔV_t^U was really zero over the period of measurement. Finally, the first-difference approach only measures the *change* in B_t.

To identify the *level* of B_t, one has to assume that B_t is equal to zero either before or after the exogenous change in leverage. This is not obviously true, especially when there is a value to interest tax shields, BDF_t, that is part of B_t.

The literature relies only on equation (7) for identification of CFD. The natural experiment approach therefore has one equation and one unknown, ΔB_t, for each observation. In contrast, I use both the value and beta relations, (7) and (8). This gives me two equations per observation. With N firms and T months of observed data, there are $2NT$ equations. These equations have to be solved for $4NT$ unknowns: the value of unlevered assets and the net benefit of debt financing (V_t^U and B_t) and their betas (β_t^U and β_t^B), for each firm-month.[5] Since it is not possible to identify $4NT$ unknowns from $2NT$ equations, I introduce two identification assumptions:

(A1) The *unlevered* asset beta, β_t^U, is either: (i) the same for some subset of firms, or; (ii) constant across time for the same firm.

(A2) Tax benefits net of costs of financial distress are a function of observable variables and the value and beta of the unlevered firm: $B_t = B(X_t)$.

Let the β_t^U vary over time but equal across the N firms, which under assumption A1(i) eliminates $(N-1)T$ unknowns. Assumption (A2) reduces the $2NT$ unknown B_t and β_t^B for each firm to a set of k parameters that determine the shape of $B(X_t)$. Together, (A1) and (A2) reduce the problem to $(N+1)T+k$ unknowns: the NT unlevered firm values, the T unlevered asset betas and k parameters. With $2NT$ equations, observing N firms over T time periods such that $(N-1)T \geq k$ allows to solve for all parameters exactly. For example, with 3 parameters in the function for B_t, it is sufficient to observe 4 firms for 1 month, or 2 firms for 3 months. A similar derivation holds for assumption A1(ii), when unlevered asset betas are constant over time but allowed to vary across firms.

To illustrate the intuition behind the identification approach, consider the following implementation for a particular industry. Assume the unlevered asset beta with the market portfolio is equal for firms within the industry, so that A1(i) is satisfied. Let the present value of the net benefit to debt financing be:

$$B_t = \left(\theta_0 + \theta_1 L_t + \theta_2 L_t^2\right) \cdot V_t^L \tag{14}$$

with leverage $L_t \equiv D_t / V_t^L$, the market value of debt divided by the total market value of the firm. The parameters θ_0, θ_1 and θ_2 are common to all firms within the industry. Since both L_t and V_t^L are observed, this specification satisfies (A2). If two companies in the same industry have the same leverage, they experience the same tax benefits and costs of financial distress (relative to firm value). The two firms must therefore have the same risk due to debt financing. Since their unlevered betas are equal by assumption, they must also have the same levered beta, β_t^L. The β_t^L of all firms in the industry then fall on the same graph against leverage, the shape of which depends on the parameters

[5] At this point I assume that the debt and equity betas are observed. The estimation of time-varying betas will be dealt with in the estimation section II.

θ_0-θ_2 alone. Estimating the levered betas of industry constituents from market values of debt and equity and fitting them against leverage thus identifies the parameters θ_0, θ_1 and θ_2.

The assumption that unlevered firms in an industry are equally risky with respect to the market portfolio is frequently used in the academic literature (e.g. Kaplan and Stein, 1990, Hecht, 2002, and implicitly in Fama and French, 1997). Practitioners also employ this assumption on a regular basis when using industry asset betas to value companies. The economic intuition behind this assumption is that firms in the same industry have the same market risk of operations. Hamada (1972) and Faff, Brooks and Kee (2002) provide some empirical support for the hypothesis that asset betas with respect to the market portfolio are the same within industries (as defined by two-digit SIC codes). Other popular risk factors such as SMB and HML (Fama and French, 1993, 1996) cannot be used because smaller firms within the industry will load higher on SMB than larger firms, and distressed firms will load higher on HML.[6]

Despite the empirical evidence, there are theoretical reasons why firms' unlevered asset betas may be related to leverage. For example, firms in economic distress have higher operating leverage and therefore higher asset betas. On the other hand, firms with higher asset betas may adopt lower leverage ratios a priori. The simulations in appendix C show that minor violations of (A1) increase the standard error of parameter estimates of the function $B(X_t)$, but do not cause severe inconsistency in the parameters, even when β_t^U is correlated with X_t (or in X_t itself).[7]

The model for B_t in (14) is a simple generalization of both the traditional no-taxes, no-CFD model (let $\theta_0 = \theta_1 = \theta_2 = 0$), and the model with tax benefits only (let $\theta_0 = 0$, $\theta_1 = \tau$ and $\theta_2 = 0$ to recover $V_t^L = V_t^U + \tau D_t$ and equation (11)). The parameter $\theta_2 \leq 0$ makes B_t curve downwards as leverage increases, and captures both the decrease in the present value of tax benefits and the costs of financial distress. Figure 2 illustrates how different values of θ_2 affect the relation between leverage and β_t^L. The next section elaborates on the economic intuition behind this specification of B_t. Section IV further discusses the interpretation of the model parameters.

A major benefit of this approach is that it can be substantially generalized to a wide class of models of the net benefit to debt financing. The intuition behind identification extends to models in which firms have different tax benefits and distress costs at the same leverage ratio. If there are other observable variables besides leverage that drive tax benefits and CFD and that are correlated with L_t, they can simply be added to the specification of B_t. In addition, the functional form of B_t can be relaxed by adding higher powers of X_t. The main limitation to the methodology is that assumption (A2) is violated when variables in B_t are unobservable.[8] The θ-estimators are inconsistent if the

[6] If there are other portfolios that unlevered returns to industry constituents load equally on, then it is possible to add more instances of equation (8). The benefit of doing so is that less data is required to identify the model parameters. Moreover, introducing more beta relations can be used to over-identify the model, when each beta relation holds in expectation (see section II). Over-identification is useful for testing model specification.

[7] It is possible to relax (A1) by adding the conditional regression equation of returns to the unlevered firm, $(V_{t+1}^U - V_t^U)/V_t^U$, on the risk factor(s). This adds an additional restriction on β_t^U that allows it to vary both over time and across firms while still identifying the system.

[8] One notable exception is the inclusion of V_t^U and β_t^U in the specification of B_t. The identification result is preserved

unobservables are correlated with any of the variables in X_t. This is the equivalent of an omitted variables problem in a standard regression, which causes the error term to be correlated with the explanatory variables. The effect of such an omitted variables problem is that the estimated parameters in $B(X_t)$ will be biased upwards (downwards) if the omitted variable is positively (negatively) correlated with X_t.

The identification argument in this section is based on the model equations holding exactly. To empirically implement the model, it is necessary to allow for error terms to the model equations. The next section discusses estimation in detail.

II Estimation

The empirical implementation in this paper estimates the following model from a panel dataset of corporate debt and equity values, for each industry separately:

$$\frac{V_{it}^U}{V_{it}^L} = 1 - \theta_0 - \theta_1 L_{it} - \theta_2 L_{it}^2 + u_{it} \tag{15}$$

$$\begin{aligned} \frac{V_{it}^U}{V_{it}^L} \cdot \beta_t^U &= \left[1 - \theta_0 - \theta_1 - \theta_2(2L_{it} - L_{it}^2)\right] \frac{D_{it}}{V_{it}^L} \cdot \beta_{it}^D \\ &\quad + \left[1 - \theta_0 + \theta_2 L_{it}^2\right] \frac{E_{it}}{V_{it}^L} \cdot \beta_{it}^E + v_{it} \end{aligned} \tag{16}$$

$$\begin{bmatrix} r_{it}^U - r_t^f \\ r_{it}^E - r_t^f \\ r_{it}^D - r_t^f \end{bmatrix} = \begin{bmatrix} \alpha_i^U \\ \alpha_i^E \\ \alpha_i^D \end{bmatrix} + \begin{bmatrix} \beta_{t-1}^U \\ \beta_{i,t-1}^E \\ \beta_{i,t-1}^D \end{bmatrix} \cdot (r_t^M - r_t^f) + \epsilon_{it} \tag{17}$$

Equations (15) and (16) are derived from a simple specification of the present value of tax benefits net of costs of financial distress for firm i at time t, B_{it}:

$$B_{it}/V_{it}^L = \theta_0 + \theta_1 L_{it} + \theta_2 L_{it}^2 + u_{it} \tag{18}$$

where the error term u_{it} is by assumption orthogonal to L_{it}. The N-by-1 vector $u_t = [u_{1t} \ldots u_{Nt}]$ is distributed i.i.d. Normal with mean zero and constant covariance matrix $R = E(u_t u_t')$.

In order to give the model a structural interpretation it is important that the error term u_{it} in equation (18) is independent of L_{it}. This assumption requires that leverage is the only variable that drives tax benefits and CFD for all firms within an industry. This is a reasonable specification if all firms within an industry have similar investment opportunities, production technology, tangibility of assets and produce similar goods or services (e.g. durable versus non-durable goods), and these characteristics are stable over time. Structural models (e.g. Merton, 1974, and Leland, 1994) then imply a one-to-one relation between L_{it} and a firms' probability of default. A company files for

since these two unobservables are already identified within the model. Adding them to B_t therefore does not introduce any new unobserved variables.

bankruptcy if the value of the unlevered firm hits the bankruptcy boundary, which depends on the firm's use of debt in its capital structure. At this point equity is worthless, i.e. $L_{it} = 1$.[9]

Recent work by Leary and Roberts (2005) reveals evidence in favor of a Trade-Off theory with adjustment costs. Economic shocks to the firm mechanically change its leverage ratio (Welch, 2004) and, by equation (18), change B_{it}. Management allows leverage to float around until the gain in market value from readjusting outweighs the cost.[10] Even though all firms within the industry have the same optimal leverage ratio, the existence of adjustment costs generates a spread in observed leverage ratios that is necessary for identification of the θ's. Fischer et al. (1989) show that even small transaction costs produce large variations in observed leverage ratios while producing a relatively small effect on optimal capital structure, compared to taxes and bankruptcy costs. Section IV shows that the observed range of leverage ratios is consistent with relatively low adjustment costs.

The assumption that u_{it} is independent of L_{it} rules out the potential simultaneity problem of L_{it} and B_{it} being jointly determined. The optimal capital structure is determined by the parameters θ_1 and θ_2 but not by u_{it}, so that u_{it} does not show up in the first-order condition for optimal leverage. Relaxing this assumption in a full-blown simultaneous equations setting that includes an equation for the observed leverage ratio requires an instrument that is correlated with L_{it} but independent of u_{it}. In a Trade-Off theory with adjustment costs model, the past return to the firm since the last refinancing determines the deviation of observed leverage ratio from its optimum, and could therefore serve as a valid instrument (Strebulaev, 2007). Such generalizations are not explored in this paper but can be easily dealt with using the proposed methodology.

In the above scenario the u_{it} represent observation errors in the market values of debt and equity, and errors in the estimation of the betas. If there are other factors besides leverage that drive CFD *within an industry*, they are subsumed by u_{it} and (18) is misspecified. If these factors are correlated with leverage, an omitted variables problem arises. In this case it is likely that the error term is positively correlated with leverage. For example, firms with high growth opportunities may have higher CFD (lower B_{it}) at the same leverage ratio than firms with few growth opportunities in the same industry. The high-growth firms will optimally choose to adopt lower leverage ratios, resulting in a positive correlation between u_{it} and L_{it}. Both θ_1 and θ_2 are then biased upwards. Costs of financial distress are under-estimated and optimal leverage, as implied by the model, is over-estimated. In this case the specification for B_t can be expanded by adding other observable variables that capture the omitted factors, allowing firms within an industry to have different CFD at the same level of financial leverage and hence, different optimal leverage ratios.

Equation (16) describes the relation between a firm's asset beta and the betas of debt and equity after B_t and its beta are substituted out. Appendix A shows how the beta of B_t can be expressed as

[9] As an example of a structural motivation for the specification of B_{it}, the Leland (1994) model implies $B_{it}/V_{it} = \theta_1 L_{it} + \theta_2 L_{it}^{X+1}$ with $\theta_1 = \frac{1+X}{X}\frac{\tau}{1-\tau}$ and $\theta_2 = -\left(\frac{1+X}{X}\frac{\tau}{1-\tau} + \alpha\right)$, where α is the loss-given-default, $X = 2r/\sigma^2$ and $L_{it} \equiv V_B/V_{it}$.

[10] Management may even be tempted to adjust away from the optimal leverage ratio to take advantage of market timing (e.g. Baker and Wurgler, 2000).

a function of the debt and equity betas.[11] Since the beta relation is derived from the value equation, the error term in equation (8) is potentially correlated with the vector u_t. As shown in equation (16), I assume an additive error $v_t = [v_{1t} \ldots v_{Nt}]$ that is distributed i.i.d Normal with mean zero and covariance matrix $S = E(v_t v_t')$, and a general contemporaneous covariance with u_t, represented in the matrix $Q = E(u_t v_t')$. If the correlation between the error terms is substantial, this will show up as large standard errors of the parameter estimates.

In the discussion of identification it was assumed that the conditional betas of debt and equity returns are observed. In reality the betas have to be estimated. The set of equations (17) augments the model with the regression equations to estimate the conditional betas with the market portfolio. I define r_t as a return from time t-1 to t. $r_t^M - r_t^f$ is the return on the market portfolio in excess of the one-month risk-free rate. Since the beta relations derived in this paper are mechanical, the regression equations in (17) do not imply that the CAPM is the true asset pricing model, and the intercepts are not required to equal zero. The regressions are merely used to calculate the necessary betas. The 3N-by-1 idiosyncratic return vector $\epsilon_t = [\epsilon_{1t} \ldots \epsilon_{Nt}]$ is orthogonal to the excess market return, and distributed i.i.d. Normal with mean zero and covariance matrix Σ. The matrix Σ is unrestricted since there is likely to be substantial cross-sectional correlation between idiosyncratic returns of debt, equity and unlevered assets of the same firm, as well as between firms within the same industry. It is also possible that ϵ_t is correlated with u_t and v_t, and the estimation will allow for that as well.

To satisfy (A1), I assume that the unlevered asset betas, β_t^U, are equal for the cross-section of firms within the same industry. The common unlevered asset beta is allowed to vary over time and follows a mean-reverting AR(1) process:

$$\beta_t^U = \phi_0 + \phi_1 \cdot \beta_{t-1}^U + \eta_t \tag{19}$$

with $|\phi_1| < 1$. Previous studies (e.g. Berk, Green and Naik, 1999) have argued that betas should be mean-reverting to ensure stationarity of returns. The AR(1) process on β_t^U, although not strictly necessary, helps to smooth the beta process so that results are more stable. The error term η_t is distributed i.i.d. Normal with mean zero and variance H, and is uncorrelated with ϵ_t.[12] It is not necessary for estimation to impose a time-series process on the equity and debt betas, but to ensure smoothness and tighter estimation bounds I run the estimation with an AR(1) on debt and equity betas, with a general correlation structure. Mean-reverting debt and equity betas are consistent with leverage being mean-reverting (see Collin-Dufresne and Goldstein, 2001, for supporting evidence). Appendix C confirms that this assumption works well in simulations, even when it is violated.

To estimate the model, one could use a relatively simple two-step procedure: 1) estimate the conditional equity and debt betas in (17), for example using rolling regressions, and; 2) estimate

[11] Avoiding the substitution of β_t^B as a function of debt and equity betas will eliminate any linearization errors in calculating β_t^B, but increases the computational burden of estimation.

[12] For equity betas one would expect a negative correlation between η_t and ϵ_t due to the leverage effect, although empirical studies do not confirm this (e.g. Braun, Nelson and Sunier, 1995). Since we are estimating *unlevered* beta there is no strong theoretical reason to assume a correlation between η_t and ϵ_t.

equations (15)-(16) using maximum likelihood, taking the point estimates of the betas as given. For an application of the first step, see for example Jostova and Philipov (2005), who use Bayesian methods to estimate stochastic betas that follow an AR(1) process. However, this procedure ignores the sampling error in the betas in the second step, which is quite substantial. Moreover, the likelihood function is difficult to derive. Integrating out the unlevered asset values and betas from the likelihood is problematic and slows down the estimation. The dimensionality of the parameter vector makes it difficult to find the maximum of the likelihood function. Finally, when using rolling regressions a sizeable number of observations have to be dropped to estimate the first betas.

I estimate the parameters of the model jointly with the conditional betas and unlevered asset values by using a Markov Chain Monte Carlo (MCMC) algorithm. This simulation-based estimation methodology is explained in detail in Robert and Casella (1999) and Johannes and Polson (2004), and in particular for structural models of the firm in Korteweg and Polson (2006). MCMC provides a way of obtaining a sample from the posterior distribution of the model's parameters and unobserved variables (the betas and unlevered asset values), given the observed values of debt and equity. Once this sample is obtained, the unobserved variables can be numerically integrated out, leaving the distribution of the parameters θ_0-θ_2, conditional on the observed data. This integration step only has to be done once. At the core of this methodology lies the Clifford-Hammersley theorem, which allows for a break-up of the joint posterior distribution of parameters, betas and unlevered asset values. Instead of drawing from the joint distribution, the theorem allows for separate draws from: i) the distribution of parameters given the betas and unlevered asset values; ii) the distribution of betas given parameters and unobserved values, and; iii) the distribution of unlevered asset values given parameters and betas. These complete conditionals are much easier to evaluate and sample from, using simple regressions and basic linear filters.

As an added bonus, MCMC provides a convenient way to deal with missing data. This is especially useful for companies with infrequently traded bonds. In essence, missing values are treated as additional model parameters. The sampling procedure automatically takes into account the uncertainty over these values, and they are integrated out in the end.

Appendix B describes in detail how a sample from the joint posterior distribution is obtained by drawing samples from the complete conditionals. Appendix C shows that the estimation methodology performs well in simulated datasets. The next section describes the sample selection procedure and provides summary statistics for the empirical application.

III Data

I construct a sample of monthly debt and equity values for firms in the National Association of Insurance Commissioners (NAIC) database over the entire coverage period 1994-2004. Insurance companies are required to file all their trades in corporate bonds with the NAIC, who makes these records available in electronic form. Hong and Warga (2000) report that insurance companies account for about 40% of all trades in the investment-grade bond market, and 25% of trades in the market for non-investment grade bonds. With over 1.3 million transactions in total, the NAIC database is the most comprehensive source of corporate bond prices currently available. The availability of transactions for non-investment grade bonds is an important feature because the market values and betas of distressed firms are especially informative for estimating costs of financial distress. Data on the amount outstanding, seniority and security of each bond was collected from the Fixed-Income Securities Database (FISD), compiled by Mergent.

From the NAIC transactions data I compute month-end bond values for each outstanding bond issue of every firm. Since not all bonds are traded every month, it is not always possible to aggregate the individual bond values to obtain the market value of all publicly traded debt. To mitigate this missing data problem I group together bonds of the same firm that are of equal security and seniority, and maturity within two years of one another. Assuming these bonds have the same interest rate and credit risk, missing values are calculated from contemporaneous market-to-book values of bonds in the same group that are observed in the same month. For those months in which none of the bonds in a group trade, the estimation algorithm integrates out the missing values of each group (see appendix B for details). The large bond issues of a firm trade more often than small issues, and I select those firms for which the largest bond groups representing at least 80% of the company's total bond face value trade at least 50% of the time. On a face-value weighted basis, table II shows that the corporate bonds in the sample trade about 71% of the time.

I also include firms from Compustat without any short or long-term debt in their capital structure. The unlevered value of these firms is directly observed, because these firms effectively have zero leverage. The addition of zero-leverage firms helps in estimating the unlevered industry beta.[13] I supplement this sample with monthly market values of equity (common plus preferred) from CRSP and accounting data from Compustat, matching companies to the FISD by their CUSIP identifier. I include the monthly dividend and interest payments in the calculation of returns to debt and equity, to control for differences in payout policy that may affect firms' unlevered asset betas.

The model is estimated on an industry-by-industry basis, defining industries by their 2-digit SIC codes. I use only those industries with data for at least two firms that have some debt at any given time, a condition required for identification. The sample comprises 269 firms in 23 industries, for a total of 22,620 firm-months. Of these firms, 235 had some debt in their capital structure over the sample period. Table I gives an overview of the 23 industries in the sample with the yearly average number of firms and equity market capitalization in each industry. On average I observe 237 firms each year, representing 6.5% of all Compustat firms in these industries. In terms of equity value the sample represents a little over 2 trillion, which is about a quarter of the market capitalization of all Compustat firms in the sample industries. The sample is biased towards larger firms, which have more actively traded bonds, but there is no bias towards more or less distressed firms.

A more troubling issue is that the market values of bank debt and capitalized leases are never observed because these securities are not publicly traded. Table II shows that on average I observe

[13]My thanks go to Ilya Strebulaev for suggesting the addition of zero-leverage firms to the sample. A previous version of this paper did not include the zero-leverage firms, with very similar empirical results.

61% of a firm's debt on a book value basis. To deal with this problem I estimate the model using two alternative assumptions for the market value of the unobserved debt: i) use the face value of the unobserved debt, and; ii) apply the minimum credit spread of the observed bond groups to the unobserved debt. I estimate the credit spread in each month from observed market values and the Nelson-Siegel (1987) model for risk-free rates, using a cubic spline to account for missing months. I then discount the face value of the unobserved debt by the two-year credit spread to approximate the market value. Since even the safest publicly traded bonds are more risky than bank debt, this provides a lower bound on the market value of the unobserved portion of debt.

Using the face value of the unobserved debt instead of the market value provides a lower bound estimate of CFD. When the firm is close to bankruptcy, its computed market value is overstated and does not drop as fast as it does in reality, resulting in under-estimated CFD. Using the credit spread of the safest bonds to calculate the market value of the unobserved debt yields an upper bound on CFD: the bank debt is considered too risky and computed firm value drops too fast when the firm is close to default.

It is important to observe a wide range of leverage ratios within each industry in order to get a clear picture of how costs of financial distress vary with leverage. Table III shows the spread of observed leverage by industry, where leverage is measured as: i) the market value of debt (net of cash) divided by the market value of assets (net of cash), and; ii) interest cover, defined as EBITDA divided by interest expense, bounded below at 0 and above at 20. By netting out cash I effectively use the market value of *operating* assets in the estimation. This counters the problem that unlevered asset betas of firms within the same industry may differ because some firms have more cash on hand than others, even if their operating asset betas are the same. On average, firms have a leverage ratio of 0.31 with a standard deviation of 0.16. Interest cover is 9.71 on average and has a standard deviation of 5.3. Both measures indicate a substantial spread in observed leverage. Table III also reports the range of credit ratings that is observed in each industry. In general, industries contain firms with credit ratings ranging from AA-AAA down to B-BB, and can go as low as D for industries such as Airlines (SIC 45) and Telecom (SIC 48).

IV Results

In this section I first examine the estimated magnitude of costs of financial distress, followed by an analysis of the characteristics that explain the variation in costs of financial distress across industries. Finally, I test the model's predictions regarding optimal capital structure.

A Costs of Financial Distress

The model specifies the present value of the net benefit to debt financing relative to the size of the firm as a quadratic function of leverage, as shown in equation (18). The posterior mean and standard deviation of the parameters θ_0, θ_1 and θ_2 for each industry are reported in table IV.[14] The parameters in this table are estimated using the face value of unobserved debt to proxy for its market value. For all industries the posterior mean of θ_1 is positive, whereas it is negative for θ_2. This result implies that the value of a company first increases as the firm takes on debt but starts to decrease when leverage becomes high, consistent with the Trade-Off theory of optimal capital structure.

The linear-quadratic model for B is a simple generalization of the Modigliani-Miller with taxes model. In general, different assumptions regarding firms' financing policies give rise to different models of the present value of tax shields. For example, a growing company that maintains a fixed debt ratio will have a value of tax shields that is different from $\tau \cdot D_t$, as shown by Miles and Ezzell (1985). Such alternative models can be estimated using the same methodology as proposed in this paper, but are not pursued here. It is therefore important to interpret the results in this section with this caveat in mind.

If the model is well-specified, the intercept term, θ_0, equals zero: when the firm has no debt ($L = 0$), tax benefits and costs of financial distress are zero ($B = 0$). The intercept θ_0 is close to zero for most industries, although it tends to be on the positive side, especially for those industries that have no zero-leverage firms in the data. This result suggests that the specification of B can be improved upon.

The mean squared errors reported in table IV show that the model fits some industries better than others. To give an impression of the fit of the model, figure 3 graphs the posterior mean of V^L/V^U versus leverage in the top panel, and the posterior mean of β^L versus leverage in the bottom panel, for each firm-month of the first industry in the sample, Oil & Gas. The estimated relationships are plotted in the same figure. The graphs clearly show the hump-shaped relation between V^L/V^U and leverage, and the U-shaped relation between β^L and leverage. The true relation between β^L and leverage is somewhat obscured by the substantial time-variation in the unlevered asset beta of this industry, which is especially low during the internet boom (see figure 4). The bottom panel of figure 3 shows the band of β^L's that is consistent with the range of β^U's over the time series.

In a world in which firms adhere to a fixed debt level, θ_1 captures the marginal tax rate that shields the first dollar of debt. The estimate of θ_1 equals 0.367 on average across industries, corresponding to a tax rate of 36.7%. This is roughly equal to the top corporate tax rate of 35% but higher than the 21.1% relative tax advantage to debt after personal taxes.[15] The positive correlation between θ_1 and industry operating profit confirms that the cross-sectional differences in θ_1 are at least partly driven by different marginal tax rates. DeAngelo and Masulis (1980) argue that non-interest tax shields

[14] An earlier version of the paper estimated the model on total return volatilities instead of betas, where volatilities follow a GARCH process. The results are substantially the same.

[15] The relative tax advantage of debt is calculated using rates from 1999: a corporate tax rate of 35%, tax on interest payments of 39.6% and 26.8% on dividends and capital gains (equal-weighted between the 14% capital gains tax rate and 39.6% rate on dividends). The numbers are from Brealey and Myers (2000, p.507). In the Leland (1994) interpretation of the results, $\theta_1 = 0.35$ corresponds to a marginal tax rate $\tau = 0.2$ (20%) for typical values of the risk-free interest rate, $r = 0.05$, and volatility of returns to the unlevered firm, $\sigma = 0.2$. This is very close to the 21.1% marginal tax rate after personal taxes.

such as depreciation can serve as a substitute for the debt tax shield. I find a strongly negative correlation between θ_1 and annual depreciation relative to sales, suggesting that the tax benefits of debt are indeed lower when earnings are shielded by depreciation (result not shown).

Graham (2000) performs a careful study of the present value of tax benefits and finds them to be 10% of firm value. However, Graham's marginal tax rates are estimated for firms that are already levered up, whereas θ_1 measures the benefit of the very first dollar of debt. Another explanation is that besides marginal tax rates, θ_1 captures non-tax benefits of debt, such as reductions in the agency costs of outside equity due to the commitment to pay out free cash flows (Jensen, 1986). These additional benefits raise the value of debt financing relative to Graham's result, who only measures marginal tax rates.

The parameter θ_2 makes B curve downward as leverage increases, and is equal to -0.683 on average, as reported in table IV. If the marginal tax rate is constant, θ_2 perfectly captures the costs of financial distress. The net benefit to debt financing, B, can then be split neatly into the gross benefit to debt financing, $BDF/V^L = \theta_1 L$, and ex-ante costs of financial distress, $CFD/V^L = -\theta_2 L^2$. If the marginal tax rate decreases as firms lever up, θ_2 represents both the decrease in marginal tax rates and the costs of financial distress. Without strong assumptions on how marginal tax rates depend on leverage it is not clear what part of θ_2 represents taxes versus CFD. The existence of non-tax benefits to debt financing raises the difficulty of separating the benefits and costs to debt financing. To resolve this issue, I calculate an upper and lower bound on the costs of financial distress. The upper bound assumes that marginal tax rates and non-tax benefits are a constant proportion of debt value so that $BDF/V^L = \theta_1 L$, as above. Any decrease in marginal tax rates as leverage increases is then absorbed by the CFD estimate, making this an effective upper bound. The lower bound assumes that firms only experience CFD when tax benefits become worthless, which happens only at very high leverage.

The upper bound on CFD is equal to $-\theta_2 L^2$. Table VI shows how CFD as a fraction of firm value depend on the leverage ratio that firms choose. For leverage ratios up to 0.3, CFD are less than 6% of firm value for most industries. When firms achieve leverage ratios of 0.5, costs of financial distress rise to an average of 17.1% of firm value. For leverage ratios higher than 0.5, average CFD grow as high as 55.3% of firm value. Firms in most industries experience CFD of up to 60% of firm value, but seven industries show even higher costs of distress as firms spiral towards default. It is likely that these extreme CFD are never observed because firms generally file for bankruptcy before equity becomes worthless ($L = 1$). At the observed leverage ratios that industries experienced over the 1994-2004 sample period, the last column in table VI shows that CFD were no more than 16.3% of firm value and equal to 4.8% on average.

The lower bound on CFD is calculated as the maximum of $-\theta_1 L - \theta_2 L^2$ and zero. The intuition is that only CFD can push the value of the firm below the value of the unlevered firm, resulting in $B < 0$. Table VII shows that the lower bound on CFD is close to zero for leverage ratios up to 0.3, and increases to an average of 24.2% as leverage approaches one. For observed levels of leverage, CFD are essentially non-existent, as no industry is levered high enough during the sample period for B to drop below zero.

The lower bound on CFD is most realistic for firms that are close to default, because the present value of tax benefits is likely close to zero (especially if firms tend to be economically distressed when filing for bankruptcy). If companies default when equity is worthless ($L = 1$), the ex-post CFD (or "loss-given-default") are $-(\theta_1 + \theta_2)$.[16] Table X shows the mean and standard deviation of ex-post CFD. The mean estimate of 26-31% of firm value is higher than the 10-23% found by Andrade and Kaplan (1998). This may be due to sample-selection in the study of Andrade and Kaplan, or their use of book values of all corporate debt, but can also be explained by the fact that firms do not wait to file for bankruptcy until equity is worthless. The four bankruptcies in the sample had market leverage (L) of 0.6-0.8 at default. If firms go bankrupt at leverage ratios of 0.7-0.9 then table VII shows that CFD at default are 11-24% of firm value.[17]

The estimates of ex-ante CFD do not distinguish between direct and indirect costs of financial distress. Warner (1977) and Weiss (1990) find that direct costs of financial distress are small, at around 3.1% of firm value. Based on direct costs of going bankrupt of 3%, the indirect costs of financial distress at default would be about 8-21%. For ex-ante CFD the difference is much less important, because the direct costs need to be multiplied by the risk-neutral probability of default to obtain their present value.

Figure 5 shows the time-series of ex-ante costs of financial distress over the 1963-2004 period, both equally-weighted and value-weighted across the sample industries. Costs of distress are calculated as in table VI, using industry leverage ratios (book value of debt over book value of debt plus market value of equity) from annual Compustat data of all firms in the sample industries. The graph shows that costs of financial distress are relatively low during booms, when most companies are doing well. When recessions hit, CFD spike up because many firms enter financial distress. After the initial spike, ex-ante CFD decrease as distressed firms either go bankrupt or refinance, resulting in a decrease in the aggregate probability of financial distress.

Estimating the model using the credit spread of each company's safest bonds to calculate the market value of its bank debt slightly increases the magnitude of estimated CFD. Table V shows that both the average θ_1 and θ_2 are a slightly lower across the two sets of estimates, because the market value of debt declines when a firm gets close to default. Table VIII shows that the upper bound on CFD is 5% of firm value for observed levels of leverage, and does not exceed 16.9% for any industry. If firms file for bankruptcy when L is in the range 0.7-0.9 then average CFD at default are 12-28% (see table IX).

The results on ex-ante CFD are consistent with Almeida and Philippon (2006), who discount Andrade and Kaplan's estimates of ex-post CFD using risk-neutral probabilities of default in a multi-period setting. They find that for investment-grade firms (with typical leverage ratios up to 0.3), CFD are between 0.2% and 6.3% of firm value and can rise up to 13.3% for a B-rated firm

[16] In the Leland (1994) interpretation, $-(\theta_1 + \theta_2)$ is exactly equal to α, the loss-given-default.

[17] A related conjecture, which is not tested here, is that firms with higher potential CFD are more likely to file for bankruptcy earlier in their decline, precisely to avoid the high CFD.

(corresponding to a typical leverage ratio of 0.42).

The results are robust to dropping the years 2003 and 2004 from the sample, in which the taxes on dividends and capital gains were significantly lower as a result of the 2003 Jobs and Growth Tax Relief Reconciliation Act. The parameter θ_1 is marginally higher when estimated over the 1994-2002 sample, and θ_2 is marginally lower, although both are in the 95% credible interval of the full-sample estimates (results not reported). The tax benefit of debt relative to equity therefore decreased only marginally due to the 2003 tax change. The change in θ_2 confirms the hypothesis that this parameter not only captures the costs of financial distress, but also incorporates a tax effect, as discussed above.

B The Cross-section of CFD

At the same leverage ratios, tables VI-IX show that the estimated ex-ante costs of financial distress are different across industries. This implies that some industries have higher *potential* CFD than others, as captured by the parameter θ_2.[18] In this section I study the industry characteristics that make firms more or less sensitive to losing value when they enter financial distress.

Strebulaev (2007) shows that the traditional regressions of observed capital structure on industry characteristics can reject the Trade-Off theory when there are adjustment costs. An advantage of the methodology in this paper is that regressing the model parameters on industry characteristics does not suffer from this problem. The model parameters capture optimal leverage and are not affected by firms' temporary deviations from the optimum.

Shleifer and Vishny (1992) argue that distressed firms may be forced to sell assets at below-market values because it is likely that their competitors, the prime candidates to buy the assets, are also distressed or bankrupt. This is especially true for intangible assets, which are not easily sold to companies outside of the industry e.g. brand names, franchises, patents and client lists. The regression results in the first column of table XI show that industries with high levels of intangibles relative to the book value of assets (value-weighted across all industry constituents in Compustat) tend to have lower θ_2. The regression coefficient is significant at the 5% level. A one-standard-deviation increase in intangibles (as a fraction of book assets) decreases θ_2 by 0.22. This corresponds to an increase in ex-post CFD of 22% if the firm files for bankruptcy when equity becomes worthless, or a 2% increase in ex-ante CFD at a leverage ratio of 0.3. Moreover, the regression coefficient on intangible assets becomes more negative as industry profitability (defined as value-weighted EBITDA divided by sales) declines. The positive interaction between intangibles and profitability is not statistically significant, but does have a large economic effect on the relation between intangibles and CFD: a one-standard deviation decrease in profitability increases the coefficient on intangibles by 0.8. This evidence is consistent with Shleifer and Vishny (1992) and Acharya et al. (2007) and shows that firms with many intangible assets lose value when distressed while industry performance is poor, and consequently have high ex-ante CFD.

[18]Observed CFD show much less variation across industries because firms in industries with high potential CFD choose lower leverage ratios. This issue is analyzed in more detail in the next section.

Theory predicts that firms with high growth opportunities will be prone to under-investment problems when they become distressed, due to a debt overhang problem (Myers, 1977). Shareholders are unwilling to fund new projects because most of the gains will go to bondholders, and bond covenants usually prohibit the firm from raising new, more senior debt. I consider two industry measures of growth opportunities: research and development (R&D) expense relative to book value of total sales and the market-to-book ratio (M/B). Both measures are value-weighted over all industry constituents in Compustat. Columns 3 and 4 of table XI show that both R&D-to-Sales and M/B are negatively related to θ_2 and significant at the 1% level. A one-standard-deviation increase in R&D-to-Sales (M/B) is associated with a decrease in θ_2 of 0.36 (0.31). The results confirm that CFD for firms industries with high growth opportunities increase faster and grow larger than in industries with few investment opportunities.

If a distressed firm relies on specialized human capital, it is susceptible to employees either leaving the firm or spending their time updating resumes and looking for another job, causing the firm to lose value. Berk et al. (2006) argue that human capital risk can be as important as taxes in determining optimal capital structure. Taking labor expense relative to either sales or cost of goods sold as a proxy for the degree to which an industry relies on human capital shows that industries with high human capital also tend to have higher potential CFD, although the regression coefficient in table XI is not statistically significant. Still, a one-standard-deviation change in the labor expense to sales ratio leads to a decrease in θ_2 of 0.16, which is equivalent to a 16% increase in ex-post CFD if the firm files for bankruptcy when equity becomes worthless, and an increase in ex-ante CFD of up to 1.4% at a leverage ratio of 0.3.

Industries that produce durable goods such as machinery and cars face the problem that customers and suppliers grow concerned about the continuity of service, warranty and parts delivery when firms approach default (Titman, 1984). I define a dummy variable that equals 1 for industries that produce machinery and equipment (SIC 35-39) and another dummy for non-financial, long-term relationship-based services (advertising, security, computer programming, data processing and healthcare (SIC 73 and 80)). The regression results show that the machinery and equipment producers have CFD at default that are 32.6% higher than other firms, although the coefficient is not statistically significant. The relationship services industries have no different CFD than other industries. The latter result may be due to existing customers being "locked in" to the relationship so that even though firms may not gain new customers, it will not lose its existing customers either. When controlling for all other above-discussed factors, the coefficient on machinery industries goes down to -0.08, suggesting there is some residual effect of the machinery industries over and above the other characteristics that explain CFD. Titman and Wessels (1988) argue that R&D plus advertising expense relative to sales also serves as a measure of product uniqueness. Including advertising expense in the measure for R&D gives similar results to R&D-to-Sales as shown in table XI.

A regression of θ_2 on liquidity in the equity market (the average number of shares traded monthly relative to shares outstanding) and the log of book assets reveals evidence that firms that are easier to refinance have slightly lower CFD. I use liquidity as a proxy for ease of refinancing, after controlling for size, because more liquid firms tend to be more transparent. When information between investors is less asymmetric, less time is spent in acquiring and checking information, and agency problems (such as stalling and haggling by interested parties) are less severe in a distressed refinancing. The percentage of debt that is held by banks (controlling for firm size) is positively related to θ_2, although statistically insignificant. A one-standard deviation increase in the percentage bank debt increases θ_2 by 0.09. Having more bank debt appears to facilitate coordination and refinancing, and therefore reduces CFD. Having more public debt issues outstanding per firm (controlling for firm size) increases CFD, but the result is not statistically or economically significant (result not shown). These findings are robust to controlling for intangibles, R&D and M/B. It appears that coordination problems in restructuring and refinancing a distressed company have a limited impact on ex-ante CFD.

The positive (but insignificant) coefficient on firm size suggests a fixed cost effect in CFD, consistent with findings by Andrade and Kaplan (1998). Industries in which firms are on average large tend to have lower CFD as a fraction of firm value, although the coefficient is not statistically significant.

The publishing industry (SIC 27) has very high intangibles but eliminating it from the regressions does not change the conclusions, although the results are slightly less significant. Using equally-weighted instead of value-weighted industry measures has no notable effect on the results.

Changing the dependent variable to the posterior mean of θ_2, estimated using the credit spread of the safest bonds (instead of the unobserved face value) to proxy for the market value of unobserved debt, produces results that are very close in significance and magnitude to the regressions in table XI. Running the same regressions on the measure of ex-post CFD, $-(\theta_1 + \theta_2)$, gives near identical results to table XI, with the exception that the coefficients on wages-to-sales becomes significant at the 10% level.

The results in this section show that the results support the model insofar that the parameter estimates capture the factors that the literature identifies as important drivers of CFD. Growth opportunities and intangibility of assets are the most important determinants of ex-ante CFD, particularly if paired with poor industry performance. Costs of financial distress are also higher when the industry relies heavily on human capital. The impaired ability to do business in distress is most costly to firms that produce durable and unique goods that require significant post-sales parts or service. Long-term, relationship-based services and financial firms are not more prone to suffering CFD. A higher percentage of bank debt in the firm's capital structure appears to lower the costs of financial distress. Other proxies for the importance of coordination problems in a distressed refinancing have little effect on CFD. Finally, there is some evidence of a fixed cost effect to CFD.

C Optimal Capital Structure

I test the Trade-Off theory of optimal capital structure in two ways. First, I regress observed industry leverage on the estimated model parameters, and second, I calculate credible intervals for optimal debt-to-assets ratios implied by the model's estimates and compare these to observed leverage ratios for each industry in the sample.

Regressions of observed leverage on the model parameters reveal whether cross-sectional differences in tax benefits and CFD have any effect on the industry's observed capital structure. If the Trade-Off theory holds, an increase in tax benefits (an increase in θ_1) results in an increase in optimal leverage. This implies a positive coefficient on θ_1. An increase in CFD (a decrease in θ_2) lowers the optimal leverage ratio so that θ_2 also has a positive sign. The regression of industry leverage (aggregate book debt over all industry constituents in Compustat divided by aggregate book debt plus market value of equity, with book debt net of cash) on θ_1 and θ_2 in table XII shows that both parameters have a positive coefficient and are significant at the 5% and 1% level, respectively, consistent with the Trade-Off theory.

Economically, the impact of θ_1 on leverage is limited: a one-standard deviation increase in θ_1 (an increase in the marginal tax rate of 12%) raises the observed leverage ratio by 0.07. This result suggests that it is difficult to empirically verify the effect of taxes on capital structure. The economic effect of θ_2 on observed leverage is much stronger than θ_1. A one-standard deviation increase in θ_2 implies a leverage ratio that is 0.12 (about a standard deviation) higher.

A more powerful test of the Trade-Off theory is to regress observed leverage ratios on the exact prediction for optimal leverage implied by the model.[19] The specification of CFD net of tax benefits as a quadratic function of leverage has a clear prediction about the leverage ratio that firms should optimally adopt. The optimal capital structure is the debt-to-assets ratio L^* that maximizes B, the present value of the benefits of debt financing net of the costs of financial distress:

$$L^* = -\frac{\theta_1}{2\theta_2} \tag{20}$$

Note that since the model estimates the net benefit of debt financing, there is no need to separate the tax benefits from the costs of financial distress in order to compute the optimal capital structure. The problem of separating benefits and costs of debt of the previous two sections is therefore not an issue here.

If companies choose their capital structures according to the model, a regression of observed leverage on the estimate of θ_1/θ_2 should yield an intercept of zero and a coefficient of -0.5, according to equation (20). Table XII shows that 49% of the variation in value-weighted debt-to-assets across industries can be explained by the posterior mean of θ_1/θ_2 alone.[20] The hypotheses that the coefficient is equal to -0.5 and that the intercept equals zero have to be rejected at the 1% and 5% level, respectively. This result suggests that industries tend to be over-levered relative to the model-implied optimum. The explanatory power of θ_1/θ_2 is robust to controlling for other factors that are traditionally used to proxy for CFD, such as intangibles, profitability and market-to-book

[19] In the presence of adjustment costs there is a range of leverage ratios that could be considered "optimal" in the sense that an adjustment to the leverage ratio that maximizes firm value does not outweigh the associated adjustment cost. Optimal leverage in this section should be interpreted as the "global" optimum leverage ratio i.e. the leverage ratio that firms would choose if they had a free ticket to pick their capital structure without incurring adjustment costs.

[20] The posterior mean of θ_1/θ_2 is different from the posterior mean of θ_1 divided by the posterior mean of θ_2, by Jensen's inequality.

ratios (see Harris and Raviv, 1991, for a summary). When considered separately, θ_1/θ_2 and M/B do equally well in explaining the cross-section of observed leverage ratios. In a regression that includes both θ_1/θ_2 and M/B (specification VI in table XII), both are statistically significant at the 1% level. This could be interpreted as θ_1/θ_2 explaining the component of observed leverage that is related to optimal leverage, and M/B capturing departures from the optimal leverage due to past performance (Welch, 2004) or market timing (Baker and Wurgler, 2000).

The regression results are nearly identical when using equally-weighted industry leverage and explanatory variables, as well as using book leverage. Using interest coverage (defined as average EBITDA divided by interest expense, value-weighted by industry) as a measure of leverage also yields the right and significant signs on θ_1, θ_2 and θ_1/θ_2, but when controlling for other variables the explanatory power of θ_1/θ_2 disappears.

Using the model parameters estimated with the credit spread of firms' safest bonds as a proxy for the market value of unobserved debt has little effect on the magnitude or significance of the regression results (not reported).

From the posterior distribution of optimal leverage one can analyze for which industries the observed leverage ratio was significantly different from the optimum over the sample period. The posterior distributions of θ_1 and θ_2 and equation (20) are used to calculate the posterior distribution of model-implied optimal debt-to-assets ratio for each industry separately. Whenever draws for θ_1 are negative, implying a negative leverage ratio, I set the leverage ratio to zero. Conversely, when draws for θ_2 are positive or when draws from the joint distribution imply $-\theta_1 > 2\theta_2$, so that predicted leverage is larger than 1, I set leverage to 1.

The box-plots in figures 6 and 7 show that for most industries the model has a strong prediction of what optimal leverage should be, as evidenced by a tight distribution of model-implied leverage ratios. The box-plots also show the *observed* value-weighted leverage ratio in the industries over the sample period. For most industries, the observed leverage ratio is close to the model's predictions. Most importantly, not many industries are underlevered compared to the model's predictions, especially when considering the estimates obtained using the credit spread of the safest bonds as a proxy for market value of unobserved debt (in figure 7).

There are only two industries for which the observed leverage ratio is considerably different from model-implied leverage: Cars (SIC 37) and Airlines (SIC 45). The car industry performed badly over the sample period, with leverage ratios of 0.4 in the mid-90s drifting up to 0.6 by 2004. Given the extensive news coverage of financial problems at Ford, General Motors, Daimler-Chrysler in the first years of the new millennium and the bankruptcy of auto-parts manufacturer Delphi in 2005, a strong case can be made for the claim that the car industry was indeed overlevered and in financial distress during the sample period, as the model claims.

The airline industry experienced bankruptcies with high frequency, especially after 2001, and it is not surprising that observed leverage is higher than optimal. The sample on which parameters are estimated contains the bankruptcy of U.S. Airways in 2002 (and again in 2004) and ATA in 2004. During the 1994-1997 period, the value-weighted leverage for the airline industry was 0.27, right around the model-implied optimum. Airlines that performed well over the sample period, such as Southwest Airlines, had even lower leverage ratios of around 0.10-0.15.

Since the car and airlines industries were overlevered over the sample period, it is not surprising that these are the two industries that had the highest ex-ante CFD estimates for the leverage ratios that were observed (16.3% and 12.9% in the last column of table VI, respectively). The high ex-ante CFD estimates can thus be explained by the high probability of default in these industries, and not because the ex-post CFD are so much different from other industries (as can be seen in table X).

There may be some concern that the results on optimal leverage are somehow driven by the estimation methodology. Even though optimality of capital structure is not assumed or in any way used in the estimation, observed leverage ratios are used to estimate the model and possibly the estimation somehow "locks on" to observed levels of leverage. However, observed leverage ratios in the sample are generally different from the leverage ratios observed for the entire industry that are used in this section. For example, the insurance industry (SIC 63) has a leverage ratio in the sample of 0.18 (see table III) but the model predicts optimal leverage to be around 0.4. The entire industry had a leverage ratio of 0.35 over the sample period, showing that the model produces estimates that imply that the entire industry is close to optimally levered even though the sample firms used for estimation are not. Simulation results (see appendix C) show that firms in the industry can have random leverage ratios over the sample period and the estimation algorithm still recovers the true parameter values and hence, estimates the right optimal leverage ratio.[21]

As a final exercise I estimate the model's implications for adjustment costs. If firms readjust their capital structure when the gain in firm value outweighs adjustment costs then the observed variation in leverage should be consistent with the size of adjustment costs. Table XIII shows the gain of adjusting leverage as implied by the model and adjustment costs calculated using fees equal to 6.5% of issue size, based on 1.1% and 5.4% issuance cost (relative to issue size) for debt and equity issues respectively (Altinkilic and Hansen, 2000). The gain of adjusting leverage is calculated as the reduction in C_t/V_t^L if firms lever back up to the optimum once L_t hits the 5th percentile of the industry distribution. The gain of relevering is 2.4% of firm value on average (median 1.8%), compared to adjustment costs of 1.3%.

The gain in firm value when firms are assumed to lever down at the 90th percentile are as large as 10%. However, companies that are extremely highly levered tend to be economically and financially distressed, and agency costs due to information asymmetries and coordination problems add to the costs of readjusting capital structure (Gilson, 1997). The potential gain in firm value can therefore far outweigh the direct issuance costs without firms levering down (see Haugen and Senbet, 1978, 1988, and Fama, 1980, for a treatment of this issue). This is not an issue at the low-leverage levels reported in table XIII.

Consistent with a trade-off between the gain of refinancing and adjustment costs, firms with

[21]Unreported simulations deliberately start all firms in the industry at leverage ratios that are too high or too low compared to the true optimum and rebalances leverage to stay in the same, sub-optimal region. The methodology still estimates the correct parameter values.

high tax benefits and high costs of financial distress adopt tighter leverage bounds: a regression of the 10-90% leverage range on θ_1 and θ_2 yields a negative coefficient on the former and a positive coefficient on the latter. This regression explains about 31% of the variance in leverage ranges, and is statistically significant at the 1% level.

The results in this section show evidence in favor of the Trade-Off theory of optimal capital structure with adjustment costs. The model for CFD net of taxes explains much of the variation in capital structure across industries even in the presence of variables commonly used to explain the cross-section of capital structures. Finally, the model suggests that the under-leverage puzzle is not as severe as usually thought, at least for firms with publicly traded debt.

V Conclusions

Costs of financial distress can be identified from the market values and betas of corporate debt and equity. Two identification assumptions are necessary: i) firms within an industry have the same (unlevered) asset beta, and; ii) the ex-ante costs of financial distress net of tax benefits are a function of observable variables.

I estimate a model in which the present value of the net benefit to debt financing is a quadratic function of leverage, with different parameters for each industry. I provide two sets of estimates with different assumptions regarding the market value of unobserved debt. I separate the benefits and costs of debt financing using two assumptions on the present value of tax benefits, and report upper and lower bounds on the costs of financial distress. For observed leverage ratios over the 1994-2004 sample period, average ex-ante CFD are 5% of firm value, and do not exceed 16.3% for any industry. If firms file for bankruptcy when equity is worthless, CFD at bankruptcy are 31% of firm value on average. For more realistic levels of leverage at default, CFD are 12-28% of firm value at bankruptcy. These results are largely consistent with previous studies of costs of financial distress.

The potential costs of financial distress that a firm suffers as it approaches default increase with industry measures of growth opportunities, the intangibility of assets, and the importance of human capital and post-sales service, warranty and parts. A higher fraction of bank debt in a firm's capital structure facilitates coordination and refinancing, lowering the costs of financial distress. Besides the importance of bank debt, there is little evidence that capital structure complexity has an effect on CFD. The empirical evidence is consistent with debt overhang, the risk of asset fire-sales, reliance on human capital and concerns of lost business as important drivers of CFD, whereas the ease of refinancing bears a much smaller effect on distress costs.

Comparing model-implied optimal leverage with observed leverage ratios reveals that most industries are close to optimally levered over the 1994-2004 sample period. The Car and Airlines industries are over-levered relative to the model's predictions, but no industry is significantly under-levered. An omitted variables problem in the specification of CFD will only make this result stronger. The under-leverage puzzle is therefore not confirmed at the industry level, although the results cannot explain why some firms refuse to take on any debt at all (e.g. Microsoft).

The empirical results show that the methodology presented in the paper is a useful tool to estimate CFD and optimal capital structure on large datasets, and the simple model gives encouraging results for future research. The empirical results validate the approach to the extent that the parameter estimates capture the factors that the literature identifies as important drivers of CFD, and the model explains a significant fraction of the cross-sectional variation in leverage ratios. The empirical model can be substantially generalized by including other proxies for the probability of default besides leverage, such as Z-scores and credit ratings, or alternative models for the tax benefit to debt financing. The variables that were found to drive CFD across industries can also be used to explain within-industry differences, as well as time-variation in CFD. Other interesting avenues are to include past firm performance, performance relative to competitors, and macro-economic determinants of CFD not explored in this paper.

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Appendix A: Proofs

Proof. The relation between a firm's betas when there are CFD.

Take first differences of $V_t^U + B_t = D_t + E_t$ and rewrite:

$$\begin{aligned}
\left(V_{t+1}^U + B_{t+1}\right) - \left(V_t^U + B_t\right) &= (D_{t+1} + E_{t+1}) - (D_t + E_t) \\
\Leftrightarrow \left(V_{t+1}^U - V_t^U\right) + (B_{t+1} - B_t) &= (D_{t+1} - D_t) + (E_{t+1} - E_t) \\
\Leftrightarrow V_t^U \cdot \frac{V_{t+1}^U - V_t^U}{V_t^U} + B_t \cdot \frac{B_{t+1} - B_t}{B_t} &= D_t \cdot \frac{D_{t+1} - D_t}{D_t} + E_t \cdot \frac{E_{t+1} - E_t}{E_t} \\
\Leftrightarrow V_t^U \cdot r_{t+1}^U + B_t \cdot r_{t+1}^B &= D_t \cdot r_{t+1}^D + E_t \cdot r_{t+1}^E \\
\Leftrightarrow V_t^U \cdot \left(r_{t+1}^U - r_{t+1}^f\right) + B_t \cdot \left(r_{t+1}^B - r_{t+1}^f\right) &= D_t \cdot \left(r_{t+1}^D - r_{t+1}^f\right) + E_t \cdot \left(r_{t+1}^E - r_{t+1}^f\right)
\end{aligned}$$

where the last equation is obtained by subtracting $V_t^L \cdot r_{t+1}^f$ from both sides, where r_{t+1}^f is the risk-free rate that applies from time t to t+1 and $V_t^L = D_t + E_t$ is the value of the levered firm. Note that the last equation holds both for total returns (including payouts such as dividends and interest), and for returns net of payouts (capital gains only). Now take conditional betas with respect to some portfolio's excess return:

$$V_t^U \cdot \beta_t^U + B_t \cdot \beta_t^B = D_t \cdot \beta_t^D + E_t \cdot \beta_t^E$$

Divide this last equation by V_t^L on both sides to obtain equation (8). ∎

Proof. $\frac{B_t}{V_t^L}\beta_t^B = \left(\theta_0 + \theta_1 + \theta_2(2L_t - L_t^2)\right) \cdot \frac{D_t}{V_t^L} \cdot \beta_t^D + \left(\theta_0 - \theta_2 L_t^2\right) \cdot \frac{E_t}{V_t^L} \cdot \beta_t^E$

Start with the specification $B_t = \left(\theta_0 + \theta_1 L_t + \theta_2 L_t^2\right) V_t^L$, with $L_t \equiv D_t/V_t^L$. Perform a first-order Taylor expansion of B_t around (D_t, E_t):

$$\begin{aligned}
B_{t+1} - B_t &\approx \frac{\partial B_t}{\partial D_t} \cdot (D_{t+1} - D_t) + \frac{\partial B_t}{\partial E_t} \cdot (E_{t+1} - E_t) \\
\Leftrightarrow B_t \cdot \frac{B_{t+1} - B_t}{B_t} &\approx \frac{\partial B_t}{\partial D_t} \cdot D_t \cdot \frac{D_{t+1} - D_t}{D_t} + \frac{\partial B_t}{\partial E_t} \cdot E_t \cdot \frac{E_{t+1} - E_t}{E_t} \\
\Leftrightarrow B_t \cdot r_{t+1}^B &\approx \frac{\partial B_t}{\partial D_t} \cdot D_t \cdot r_{t+1}^D + \frac{\partial B_t}{\partial E_t} \cdot E_t \cdot r_{t+1}^E
\end{aligned}$$

Take the conditional covariance with the excess return to the risk factor from time t to t+1, $r_{t+1}^M - r_{t+1}^f$, on both sides:

$$\begin{aligned}
B_t \cdot Cov_t\left(r_{t+1}^B, r_{t+1}^M - r_{t+1}^f\right) &\approx \frac{\partial B_t}{\partial D_t} \cdot D_t \cdot Cov_t\left(r_{t+1}^D, r_{t+1}^M - r_{t+1}^f\right) \\
&\quad + \frac{\partial B_t}{\partial E_t} \cdot E_t \cdot Cov_t\left(r_{t+1}^E, r_{t+1}^M - r_{t+1}^f\right) \\
\Leftrightarrow \frac{B_t}{V_t^L}\beta_t^B &\approx \frac{\partial B_t}{\partial D_t} \cdot \frac{D_t}{V_t^L} \cdot \beta_t^D + \frac{\partial B_t}{\partial E_t} \cdot \frac{E_t}{V_t^L} \cdot \beta_t^E
\end{aligned}$$

To get the last equality, divide both sides by $Var_t(r_{t+1}^M - r_{t+1}^f)$ and V_t^L. Note that the betas here are betas on plain (not excess) returns. Subtracting the risk-free rate from r_{t+1}^B is not equivalent with subtracting the risk-free rate from both r_{t+1}^D and r_{t+1}^E, but this is not a big issue given the small effect of using plain versus excess returns when estimating debt and equity betas. Alternatively, it is possible to estimate the model on plain (instead of excess) returns, since the other model equations hold both for plain and excess returns.

Plugging the derivatives $\frac{\partial B_t}{\partial D_t} = \theta_0 + \theta_1 + \theta_2(2L_t - L_t^2)$ and $\frac{\partial C_t}{\partial E_t} = \theta_0 - \theta_2 L_t^2$ into the equation completes the proof. Note that the error term u_t in (18) will show up in the partial derivatives, but since it is by assumption independent of L_t, the additional error term that shows up in $B_t/V_t^L \cdot \beta_t^B$ is also uncorrelated with leverage. Combining the resulting expression with (8) yields (16). ∎

Appendix B: MCMC Algorithm

For a general description of Markov Chain Monte Carlo (MCMC) methods, see section II in the main text. Details can be found in Gelfand and Smith (1990), Robert and Cassella (1999), and in particular for financial economics in Johannes and Polson (2004).

The algorithm estimates the model in (15)-(17) by sampling from the posterior distribution of model parameters, betas and (unobserved) unlevered asset values conditional on observed debt and equity values, by following four main steps:

1. Draw the model parameters given the betas and values
2. Draw the betas, given values and the new parameters
3. Draw values (including simulating missing debt data), given the new betas and parameters.
4. Go back to step 1, using the new values and betas.

After a number of runs known as the "burn-in" phase, this algorithm converges to a stationary chain. This usually takes only 20-30 steps, but to be sure I let it go for 100 cycles. At this point I let the algorithm run for another 5,000 cycles (1,000 for the simulation study). These samples constitute random draws from the posterior distribution. From here it is easy to compute descriptive statistics on the parameters of interest (θ_0-θ_2), such as calculating (posterior) means and standard deviations.

To initialize the algorithm, let the θ's equal zero so that $V_{it}^U = D_{it} + E_{it}$ for all i and t. The betas are initialized from 24-month rolling regressions. Then go to step 1 and let the procedure do the work. Let's consider each step in detail.

Step 1: Draw the model parameters given the betas and values

Step 1A: Draw ϕ_0, ϕ_1 and H, given the betas

Imposing an AR(1) on all betas, estimate the parameters using a Bayesian Seemingly Unrelated Regression (see Zellner, 1971, p.240-243):

$$\begin{bmatrix} y^U \\ y_1^E \\ y_1^D \\ \vdots \\ y_N^E \\ y_N^D \end{bmatrix} = \begin{bmatrix} X^U & & & & & \\ & X_1^E & & & & \\ & & X_1^D & & & \\ & & & \ddots & & \\ & & & & X_N^E & \\ & & & & & X_N^D \end{bmatrix} \cdot \begin{bmatrix} \Phi^U \\ \Phi_1^E \\ \Phi_1^D \\ \vdots \\ \Phi_N^E \\ \Phi_N^D \end{bmatrix} + \begin{bmatrix} \eta_1^U \\ \eta_1^E \\ \eta_1^D \\ \vdots \\ \eta_N^E \\ \eta_N^D \end{bmatrix}$$

where

$$y^U = \begin{bmatrix} \beta_2^U \\ \vdots \\ \beta_T^U \end{bmatrix} \quad , \quad X^U = \begin{bmatrix} 1 & \beta_1^U \\ \vdots & \vdots \\ 1 & \beta_{T-1}^U \end{bmatrix} \quad , \quad \Phi^U = \begin{bmatrix} \phi_0^U \\ \phi_1^U \end{bmatrix} \quad , \quad E(\eta \cdot \eta') = H \otimes I_{T-1}$$

and analogous definitions for equity and debt betas of firm $i = 1 \ldots N$ (y_i^E, X_i^E, Φ_i^E and y_i^D, X_i^D, Φ_i^D). The error structure implies that the errors are i.i.d. over time but contemporaneously correlated, with covariance matrix H. Using a diffuse prior $p(\Phi, H^{-1}) \propto |H^{-1}|^{-(N+1)}$, the posterior distributions is:

$$\begin{aligned} \Phi | H^{-1}, y, X &\sim N\left[(X'(H^{-1} \otimes I_{T-1})X)^{-1} \cdot X'(H^{-1} \otimes I_{T-1})y, (X'(H^{-1} \otimes I_{T-1})X)^{-1}\right] \\ H | \Phi, y, X &\sim IW(A \ , \ T-1 \ , \ 2N+1) \end{aligned}$$

where IW stands for Inverse Wishart with characteristic matrix A and T-1 degrees of freedom. The matrix A is the symmetric, 2N+1 by 2N+1 dimensional matrix with sums of products of the residuals of the individual regressions as the elements (see Zellner, 1971, p.241).

Given the previous sample $(\Phi^{(g)}, H^{(g)}, \beta^{(g)})$, first draw $\Phi^{(g+1)}|H^{(g)}, y^{(g)}, X^{(g)}$ from the Multivariate Normal distribution, and then $H^{(g+1)}|\Phi^{(g+1)}, y^{(g)}, X^{(g)}$ from the above Inverse Wishart.

Step 1B: Draw the α's and Σ, given the betas and values

The return regression equations can be used to estimate the intercepts α and the covariance matrix of idiosyncratic noise Σ, including the covariances with v_{it}. This is done using a Bayesian multivariate regression (Zellner, 1971, p.224-228):

$$[\ y_1 \quad y_2 \quad \ldots \quad y_N\ ] = 1_{T-1} \cdot [\ \alpha_1 \quad \alpha_2 \quad \ldots \quad \alpha_N\ ] + [\ e_1 \quad e_2 \quad \ldots \quad e_N\ ]$$

where 1_{T-1} is a column vector of length T-1, filled with ones, and the 1-by-4 row vector $\alpha_i = [\ \alpha_i^U \quad \alpha_i^E \quad \alpha_i^D \quad 0\ ]$. Furthermore:

$$y_i = \begin{bmatrix} \widetilde{r_{i2}^U} - \widetilde{r_2^M}\beta_1^U & \widetilde{r_{i2}^E} - \widetilde{r_2^M}\beta_{i1}^E & \widetilde{r_{i2}^D} - \widetilde{r_2^M}\beta_{i1}^D & \frac{V_{i2}^U}{V_{i2}^L}\cdot\beta_2^U - \frac{E_{i2}^*}{V_{i2}^L}\cdot\beta_{i2}^E - \frac{D_{i2}^*}{V_{i2}^L}\cdot\beta_{i2}^D \\ \vdots & \vdots & \vdots & \vdots \\ \widetilde{r_{i,T}^U} - \widetilde{r_T^M}\beta_{T-1}^U & \widetilde{r_{i,T}^E} - \widetilde{r_T^M}\beta_{i,T-1}^E & \widetilde{r_{i,T}^D} - \widetilde{r_T^M}\beta_{i,T-1}^D & \frac{V_{iT}^U}{V_{iT}^L}\cdot\beta_T^U - \frac{E_{iT}^*}{V_{iT}^L}\cdot\beta_{iT}^E - \frac{D_{iT}^*}{V_{iT}^L}\cdot\beta_{iT}^D \end{bmatrix}$$

$$e_i = \begin{bmatrix} \epsilon_{i2}^U & \epsilon_{i2}^E & \epsilon_{i2}^D & v_{i2} \\ \vdots & \vdots & \vdots & \vdots \\ \epsilon_{iT}^U & \epsilon_{iT}^E & \epsilon_{iT}^D & v_{iT} \end{bmatrix} \quad , \qquad E(e' \times e) = \Sigma$$

defining $D_{it}^* \equiv D_{it}[1 - \theta_0 - \theta_1 - \theta_2(2L_{it} - L_{it}^2)]$ and $E_{it}^* \equiv E_{it}[1 - \theta_0 + \theta_2 L_{it}^2]$. Returns from time t-1 to t, in excess of the risk-free rate, are denoted by $\widetilde{r_{it}}$.

Using the diffuse prior $p(\alpha, \Sigma) \propto |\Sigma|^{-(4N+1)/2}$, the α's are distributed Multivariate Normal, conditional on Σ and the data:

$$\alpha|\Sigma, y \sim N(\ 1_{T-1}' \cdot y/(T-1) \quad , \quad \Sigma/(T-1)\ )$$

Not surprisingly, this is basically the distribution of a sample average. The posterior of Σ is an Inverse Wishart distribution:

$$\Sigma|y \sim IW(A\ ,\ T-2\ ,\ 4N)$$

where A is the 4N-by-4N matrix containing the sums of products of the OLS residuals from the individual regressions. The strategy then is to first draw $\Sigma^{(g+1)}|y^{(g)}$, and then $\alpha^{(g+1)}|\Sigma^{(g+1)}, y^{(g)}$.

Step 1C: Draw θ's, R, S, and Q, given the betas and values

Another Bayesian regression provides the posterior distributions of the θ's, and the covariance matrices R, S, and Q:

$$\begin{bmatrix} y_1^A \\ \vdots \\ y_N^A \\ y_1^B \\ \vdots \\ y_N^B \end{bmatrix} = \begin{bmatrix} X_1^A \\ \vdots \\ X_N^A \\ X_1^B \\ \vdots \\ X_N^B \end{bmatrix} \cdot \begin{bmatrix} \theta_0 \\ \theta_1 \\ \theta_2 \end{bmatrix} + \begin{bmatrix} u_1 \\ \vdots \\ u_N \\ v_1 \\ \vdots \\ v_N \end{bmatrix}$$

where

$$
\begin{aligned}
y_i^A &= \begin{bmatrix} V_{i1}^U/V_{i1}^L - 1 \\ \vdots \\ V_{iT}^U/V_{iT}^L - 1 \end{bmatrix} \quad , \quad X_i^A = \begin{bmatrix} -1 & -L_{i1} & -L_{i1}^2 \\ \vdots & \vdots & \vdots \\ -1 & -L_{iT} & -L_{iT}^2 \end{bmatrix} \\
y_i^B &= \begin{bmatrix} \frac{V_{i1}^U}{V_{i1}^L}\beta_1^U - \frac{D_{i1}}{V_{i1}^L}\beta_{i1}^D - \frac{E_{i1}}{V_{i1}^L}\beta_{i1}^E \\ \vdots \\ \frac{V_{iT}^U}{V_{iT}^L}\beta_T^U - \frac{D_{iT}}{V_{iT}^L}\beta_{iT}^D - \frac{E_{iT}}{V_{iT}^L}\beta_{iT}^E \end{bmatrix} \quad , \quad u_i = \begin{bmatrix} u_{i1} \\ \vdots \\ u_{iT} \end{bmatrix} \quad , \quad v_i = \begin{bmatrix} v_{i1} \\ \vdots \\ v_{iT} \end{bmatrix} \\
E & \left(\begin{bmatrix} u_i \\ v_i \end{bmatrix} \cdot \begin{bmatrix} u_i' & v_i' \end{bmatrix} \right) = W \otimes I_T \quad , \quad W = \begin{bmatrix} R & Q \\ Q' & S \end{bmatrix} \\
X_i^B &= \begin{bmatrix} -L_{i1}\beta_{i1}^D - (1-L_{i1})\beta_{i1}^E & -L_{i1}\beta_{i1}^D & -L_{i1}^2(2-L_{i1})\beta_{i1}^D + L_{i1}^2(1-L_{i1})\beta_{i1}^E \\ \vdots & \vdots & \vdots \\ -L_{iT}\beta_{iT}^D - (1-L_{iT})\beta_{iT}^E & -L_{iT}\beta_{iT}^D & -L_{iT}^2(2-L_{iT})\beta_{iT}^D + L_{iT}^2(1-L_{iT})\beta_{iT}^E \end{bmatrix}
\end{aligned}
$$

Using a similar diffuse prior as before, $p(\Theta, W) \propto |W|^{-(2NT+1)/2}$, the posteriors are:

$$
\begin{aligned}
\Theta|W, y, X &\sim N\left[(X'(W^{-1} \otimes I_T)X)^{-1} \cdot X'(W^{-1} \otimes I_T)y, (X'(W^{-1} \otimes I_T)X)^{-1}\right] \\
W|\Theta, y, X &\sim IW(A, T, 2N)
\end{aligned}
$$

where A is calculated as before. First sample $\Theta^{(g+1)}|W^{(g)}, y^{(g)}, X^{(g)}$ from the Multivariate Normal and then use the Inverse Wishart to draw from $W^{(g+1)}|\Theta^{(g+1)}, y^{(g)}, X^{(g)}$.

Step 2: Draw the betas, given values and parameters

This step requires two sub-steps: i) Use the Kalman filter to get the distribution of the betas at each time t, given the values up to time t, and; ii) sample backwards to get a draw from the distribution of the betas given all values from time 1 to T. This procedure is called Forward Filter Backward Sample (FFBS) and is described in detail in Carter and Kohn (1994).

For the Kalman filter, the state diffusion is:

$$
\begin{bmatrix} \beta_{t+1}^U \\ \beta_{1,t+1}^E \\ \beta_{1,t+1}^D \\ \vdots \\ \beta_{N,t+1}^E \\ \beta_{N,t+1}^D \end{bmatrix} = \begin{bmatrix} \phi_0^U \\ \phi_{01}^E \\ \phi_{01}^D \\ \vdots \\ \phi_{0N}^E \\ \phi_{0N}^D \end{bmatrix} + \begin{bmatrix} \phi_1^U & & & & & \\ & \phi_{11}^E & & & & \\ & & \phi_{11}^D & & & \\ & & & \ddots & & \\ & & & & \phi_{1N}^E & \\ & & & & & \phi_{1N}^D \end{bmatrix} \cdot \begin{bmatrix} \beta_t^U \\ \beta_{1,t}^E \\ \beta_{1,t}^D \\ \vdots \\ \beta_{N,t}^E \\ \beta_{N,t}^D \end{bmatrix} + \begin{bmatrix} \eta_{1,t+1}^U \\ \eta_{1,t+1}^E \\ \eta_{1,t+1}^D \\ \vdots \\ \eta_{N,t+1}^E \\ \eta_{N,t+1}^D \end{bmatrix}
$$

and the observation equations:

$$\begin{bmatrix} \widetilde{r^U_{1,t+1}} \\ \widetilde{r^E_{1,t+1}} \\ \widetilde{r^D_{1,t+1}} \\ 0 \\ \vdots \\ \widetilde{r^U_{N,t+1}} \\ \widetilde{r^E_{N,t+1}} \\ \widetilde{r^D_{N,t+1}} \\ 0 \end{bmatrix} = \begin{bmatrix} \alpha^U_1 \\ \alpha^E_1 \\ \alpha^D_1 \\ 0 \\ \vdots \\ \alpha^U_N \\ \alpha^E_N \\ \alpha^D_N \\ 0 \end{bmatrix} + \begin{bmatrix} \widetilde{r^M_{t+1}} & & & & & \\ & \widetilde{r^M_{t+1}} & & & & \\ & & \widetilde{r^M_{t+1}} & & & \\ -\frac{V^U_{1t}}{V^L_{1t}} & \frac{E^*_{1t}}{V^L_{1t}} & \frac{D^*_{1t}}{V^L_{1t}} & & & \\ \vdots & & & \ddots & & \\ \widetilde{r^M_{t+1}} & & & & & \\ & & & & \widetilde{r^M_{t+1}} & \\ & & & & & \widetilde{r^M_{t+1}} \\ -\frac{V^U_{Nt}}{V^L_{Nt}} & & & & \frac{E^*_{Nt}}{V^L_{Nt}} & \frac{D^*_{Nt}}{V^L_{Nt}} \end{bmatrix} \cdot \begin{bmatrix} \beta^U_t \\ \beta^E_{1,t} \\ \beta^D_{1,t} \\ \vdots \\ \beta^E_{N,t} \\ \beta^D_{N,t} \end{bmatrix} + \begin{bmatrix} \epsilon^U_{1,t+1} \\ \epsilon^E_{1,t+1} \\ \epsilon^D_{1,t+1} \\ v_{1,t+1} \\ \vdots \\ \epsilon^U_{N,t+1} \\ \epsilon^E_{N,t+1} \\ \epsilon^D_{N,t+1} \\ v_{N,t+1} \end{bmatrix}$$

where D^*_{it} and E^*_{it} are as defined in step 1B. Given the estimated parameters and the covariance matrices $H^{(g+1)}$ from step 1A and $\Sigma^{(g+1)}$ from 1B, this is a straightforward application of the Kalman Filter.

Step 3: Draw values, given the betas and parameters

Obtaining a sample of the unlevered asset values for all firms and time periods requires another application of FFBS. The state diffusion is:

$$\begin{bmatrix} V^U_{1,t+1} \\ \vdots \\ V^U_{N,t+1} \end{bmatrix} = \begin{bmatrix} 1 + r^f_{t+1} + \alpha^U_1 + \beta^U_t \cdot \widetilde{r^M_{t+1}} & & \\ & \ddots & \\ & & 1 + r^f_{t+1} + \alpha^U_N + \beta^U_t \cdot \widetilde{r^M_{t+1}} \end{bmatrix} \cdot \begin{bmatrix} V^U_{1t} \\ \vdots \\ V^U_{Nt} \end{bmatrix} + \begin{bmatrix} V^U_{1t} \cdot \epsilon^U_{1,t+1} \\ \vdots \\ V^U_{Nt} \cdot \epsilon^U_{N,t+1} \end{bmatrix}$$

with observations:

$$\begin{bmatrix} 1 - \theta_0 - \theta_1 L_{1t} - \theta_2 L^2_{1t} \\ \vdots \\ 1 - \theta_0 - \theta_1 L_{Nt} - \theta_2 L^2_{Nt} \\ \frac{D^*_{1t}}{V^L_{1t}} \cdot \beta^D_{1t} + \frac{E^*_{1t}}{V^L_{1t}} \cdot \beta^E_{1t} \\ \vdots \\ \frac{D^*_{Nt}}{V^L_{Nt}} \cdot \beta^D_{Nt} + \frac{E^*_{Nt}}{V^L_{Nt}} \cdot \beta^E_{Nt} \end{bmatrix} = \begin{bmatrix} 1/V^L_{1t} & & \\ & \ddots & \\ & & 1/V^L_{Nt} \\ \beta^U_t/V^L_{1t} & & \\ & \ddots & \\ & & \beta^U_t/V^L_{Nt} \end{bmatrix} \cdot \begin{bmatrix} V^U_{1t} \\ \vdots \\ V^U_{Nt} \end{bmatrix} - \begin{bmatrix} u_{1t} \\ \vdots \\ u_{Nt} \\ v_{1t} \\ \vdots \\ v_{Nt} \end{bmatrix}$$

Conditional on the parameter estimates and the covariance matrices $\Sigma^{(g+1)}$ and $W^{(g+1)}$, Kalman filter the unlevered asset values and then sample backward as in Carter and Kohn (1994).

In case there is missing debt data, simulate the missing values in every cycle of the algorithm. First group together the bonds of each single firm that are of equal security and seniority, and have maturities within 2 years of each other. Assume that the bonds within each group have the same market-to-book ratio, calculate the value of the other bonds in the group from an observed trade in one of the bonds. To fill in the time-series gaps for each group, use FFBS to draw missing values from the market model. In the simulation study there is effectively only one group of bonds, so that either all bond values are observed or completely missing. The estimation strategy is the same as above.[22]

Appendix C: Simulation and Robustness

Simulations are a useful tool to validate the performance of an estimation algorithm and to judge the sensitivity of the results to key assumptions. Specifically, I look at the effects of infrequent bond trading, unobserved bank debt and different unlevered betas across firms on the bias and standard errors of the results.

The first issue, infrequent trading of corporate bonds, results in more noisy estimation of debt betas and larger standard errors of parameter estimates, but does not lead to inconsistent estimators. The second issue, the fact that the market value of bank debt is never observed, creates a modest bias in the estimates. Under the working assumption that bank debt always trades at face value, costs of financial distress are underestimated. The reason is that when the firm approaches default, book values of bank debt overstate the value of the firm and understate its riskiness i.e. the firm is measured to be too safe so that costs of financial distress are under-estimated. The opposite happens when bank debt is assumed to have the same credit risk as publicly traded bonds. Finally, the assumption that the unlevered asset beta is the same for all firms within the same industry may be wrong. If firms with high financial leverage in fact have low unlevered asset betas, then the estimated costs of financial distress are biased downwards. The estimated common asset beta is too high for highly levered firms so that the difference between the observed levered firm beta, β_t^L, and unlevered asset beta is too low.

The simulation is set up as follows. There are 4 firms in the industry, of random initial size and financial leverage.[23] Debt and equity market values for the first month are drawn at random from a uniform distribution between 0 and 100. The unlevered firm value is then determined from the value equation (15):

$$V_{it}^U = V_{it}^L \cdot (1 - \theta_0 - \theta_1 L_{it} - \theta_2 L_{it}^2 + u_{it}) \qquad (21)$$

for each firm $i = 1 \ldots 4$. The simulation parameters are summarized in table XIV. The parameter θ_0 equals 0 so that costs of financial distress are zero when there is no debt in the capital structure of the firm. The marginal tax rate, θ_1, is chosen to be 0.2 and the CFD parameter θ_2 is set equal to -0.3. These numbers imply a marginal tax rate (including disciplinary benefits of taking on debt)

[22] Ideally, one would treat each bond issue separately so that in estimation each individual bond's beta is estimated and used to generate the missing values. Although in theory this is not a problem to implement, the multitude of bond issues per firm and computing power constraints makes this approach infeasible.

[23] At the minimum 2 firms are needed to identify the model (see section I). I checked this condition in the simulations: when estimating the model on a time series for one firm, the estimation produces random numbers. With 2 or more firms, it clearly converges to the correct parameter values.

of 20% and an optimal leverage ratio $L^* = -0.2/(2 \cdot -0.3) = 0.33$. The noise component u_{it} follows an i.i.d. Normal distribution with mean zero and variance $R = 0.005^2$. Since in this specification, CFD are -3.3% of firm value at the minimum and 10% at the maximum, a standard deviation of u_{it} of 0.5% is quite substantial. For simplicity u_{it} is drawn independent from u_{jt} for $i \neq j$, even though the estimation algorithm allows the error to be correlated across firms.

Companies' common (unlevered) asset beta equals 1 for the first month and progresses according to an AR(1) process with intercept 0.02 and autoregressive coefficient 0.98. The noise component is distributed Normal with mean zero and monthly standard deviation 0.01. This implies that asset betas are highly autocorrelated with a long-run mean of 1, and are not very volatile. Given the unlevered asset beta and current values of debt, equity and unlevered assets, we can calculate the debt and equity betas that drive expected returns over the next month:

$$\beta_{it}^{D} = 1.3 \cdot \beta_{t}^{U} \cdot (L_{it})^{5} \tag{22}$$

$$\beta_{it}^{E} = \frac{V_{it}^{U} \cdot \beta_{t}^{U} - V_{it}^{L} \cdot v_{it} - \left[1 - \theta_0 - \theta_1 - \theta_2(2L_{it} - L_{it}^2)\right] D_{it} \cdot \beta_{it}^{D}}{E_{it}\left[1 - \theta_0 + \theta_2 L_{it}^2\right]} \tag{23}$$

The debt beta will be close to zero for low levels of financial leverage and rises exponentially to equal the asset beta at default (when leverage is 1). The asset beta at default is assumed to be 30% higher than the unlevered asset beta.[24] The equity beta is calculated by rewriting equation (16), including the error term v_{it} to reflect that the beta relation is subject to noise. The error v_{it} is distributed i.i.d. Normal with zero mean and variance $S = 0.05^2$, and is independent across firms (although the estimation allows for cross-sectional dependence). The errors u_{it} and v_{it} are assumed to have a correlation coefficient of $\rho = 0.1$ for the same firm (this is a representative number for the estimates from the real data).

Using the values and betas of debt and equity in a particular month, I compute values for the next month by drawing debt and equity returns from the CAPM. The expected annual excess return on the market portfolio is 7%, with a standard deviation of 15%. The idiosyncratic shock to debt and equity returns is distributed multivariate Normal with mean zero and annual standard deviation of 10% for equity and 5% for debt, with correlation coefficient 0.3. This results in realistic levels of return volatilities: for a firm with a beta equal to 1, annual return volatility is about 25%. With betas rising to 2 or 3 when leverage ratios go up, return volatility can easily exceed 50% per year. For simplicity, I assume that idiosyncratic returns are uncorrelated across firms, although this is not required for the estimation.

With next month's debt and equity values, the new unlevered asset value and betas are calculated as above. To generate a time series of debt and equity values that can be used for estimation, I repeat this exercise as many times as desired. Consistent with the empirical data, I simulate monthly data for 11 years, or 132 months.

[24] The theory only states how levered firm beta changes as a function of leverage, but not the individual debt and equity betas. Therefore I can specify my own functions for debt and equity betas, as long as the relation (8) holds in expectation. It is not necessary to know these processes for estimation.

Base Case: Perfectly Observed Debt

The simulation is repeated 100 times using the same parameters. Each run of the simulation represents a different observed dataset of a 4-firm industry over 11 years. The simulated monthly debt and equity values are used as input for the estimation algorithm described in section II. The algorithm is initialized the same way as in the empirical application: start with all θ's equal to zero and use 24-month rolling regressions to initialize the debt and equity betas (see also appendix B). Although the algorithm usually "burns in" in around 20 cycles, I discard the first 100, and use another 1,000 cycles for each estimation.

Table XV panel A shows the estimation results when the market value of corporate debt is observed without bias. The posterior mean estimates are close to the true values, and the standard deviation of the posterior distribution is generally quite small. For example, using the posterior mean as the point estimate for θ_2 we find that over the 100 sample datasets, the estimate averages to -0.285 with a standard deviation of 0.020. The posterior standard deviation is 0.038, on average. As can be expected, for 4 out of 100 datasets the (2.5%, 97.5%) credible interval for θ_2 does not contain the true value of -0.3.

The fact that we do not observe B_{it} adds substantial noise to the estimates: if one could observe CFD directly and regress B_{it}/V_{it}^{L} on leverage, we would estimate θ_2 with a standard error of 0.005, compared to the posterior standard deviation of 0.038 from the MCMC estimation. Still, the estimates are tight enough to be meaningful. For example, the (2.5%, 97.5%) credible interval for implied optimal financial leverage is between 0.326 and 0.372, on average across datasets.

Note that equations (22)-(23) imply that using an AR(1) process on the debt and equity betas in the estimation is misspecified. However, since the expected change in leverage is usually small it does not materially affect the results. Other specifications that make the time series process of the debt and equity betas dependent on leverage do not materially change the results. Finally, the algorithm works equally well for different values of the θ's in simulation (results not reported).

Missing Debt Data

To simulate the empirical issue of missing corporate debt data due to infrequent trading, I drop half of the debt observations by imposing a 50% chance of observing the value of corporate debt in each month. The months in which debt value is missing are different for each dataset. The results in table XV panel B show that this issue does not bias the estimates but the uncertainty about the unobserved values does increase the posterior standard deviation. The variance of the estimates does not increase dramatically, due to the fact that debt values and betas are quite stable over time so that the simulated missing values are close to the true values.

Unobserved Bank Debt

A tricky issue is the fact that bank debt is generally unobserved and requires assumptions about its market value. One solution is to use the face value of bank debt as the upper bound on its market value and the market-to-book ratio of traded bonds as the lower bound. Since the mispricing that results from these assumptions is related to leverage, it is likely that the estimates of costs of financial distress are biased. To simulate this issue, I assume observations on D_{it}^{*}, which is related to the true market value of corporate debt, D_{it}, as follows:

$$D_{it}^{*} = D_{it} \cdot [1 + min(L_{it} - 0.5, 0)^2] \tag{24}$$

Debt values are observed correctly when leverage is below 0.5, but overstated up to 25% when leverage increases beyond 0.5. This is similar to using book values of bank debt instead of market values.

Table XV panel C shows that indeed θ_2 is overestimated, at -0.245 on average, but the true value of -0.3 is still within two standard deviations of the mean. For firms with high levels of financial leverage this can understate ex-post CFD by up to 4% of firm value relative to the results in panels A and B. Still, this is relatively small compared to the 25% mispricing of debt, partly because some of the mismeasurement is absorbed by the estimates of unlevered firm value, V_{it}^{U}, mitigating the bias in the parameters. If debt is undervalued by up to 25% (as if applying credit spreads of corporate bonds to bank debt), θ_2 is under-estimated at similar magnitudes.

Finally, much of the bias results from the most extremely levered firms because their debt is the most mismeasured. When I exclude those firm-months in which $L_{it} > 0.9$ the bias virtually disappears (results not shown).

Different Unlevered Betas

The main identification assumption made in section I is that firms within the same industry have the same unlevered asset beta. However, it may be the case that some firms are more levered because they face lower business risk, i.e. the assumption that the unlevered asset beta is the same for all firms within the industry may be wrong. This problem could result in underestimation of the costs of financial distress because the asset betas for highly levered firms are over-estimated.[25] To check this I simulate data with perfectly observed debt, but firms' unlevered betas at time 0 depend on their financial leverage:

$$\beta_{i0}^{U} = 1.2 - L_{i0} * 0.4 \tag{25}$$

Firms with low financial leverage have asset betas close to 1.2 whereas firms with very high leverage have asset betas close to 0.8. Table XV panel D shows that the estimates are not biased, but the posterior standard deviation does increase relative to panel A. The results appear quite robust to reasonable violations of assumption (A1), that unlevered betas across firms within the same industry are equal.

[25] Alternatively, highly levered, distressed firms may have higher asset betas as their operating leverage increases, resulting in overestimation of costs of financial distress.

Table I

Sample Breakdown by Industry

Breakdown of the 1994-2004 sample of 269 U.S. firms into industries, by 2-digit SIC code. The third column shows the average number of sample firms per year, and as a percentage of all Compustat firms in the same industry in column four. Column five shows the average number of sample firm per year that had some debt in their capital structure, and as a percentage of the industry in column six. Column seven lists the average industry equity market capitalization (in billions of dollars) in the sample, and as a percentage of total market capitalization in the industry in column eight. The final row shows the average number of sample firms and average market capitalization per year, aggregated across all industries in the sample. Source: FISD, CRSP, Compustat.

Industry	2-digit	#firms		#firms w/ debt		market cap	
	SIC	sample	%total	sample	%total	sample	%total
Oil & Gas	13	12.91	6.11	10.36	4.95	34.84	14.33
Builders	15	4.36	9.43	4.00	8.78	5.61	25.04
Food	20	10.91	7.66	8.91	6.27	97.25	16.69
Paper	26	6.00	8.80	6.00	8.80	30.24	17.95
Publishing	27	7.27	8.70	6.00	7.21	29.85	24.30
Chemicals	28	27.64	5.26	19.91	3.79	475.66	25.63
Petroleum Products	29	3.73	9.30	3.73	9.30	9.02	1.00
Primary Metals	33	4.91	4.74	4.91	4.74	62.04	59.57
Machinery	35	13.64	3.31	12.27	2.99	79.32	9.84
Electric Equipment	36	17.27	3.31	15.82	3.03	142.79	12.61
Cars	37	8.27	5.98	7.55	5.48	75.06	14.86
Instruments	38	7.55	1.84	6.55	1.59	77.44	23.52
Transport (Air)	45	4.82	10.28	4.82	10.28	25.65	46.23
Telecom	48	15.82	5.11	14.73	4.74	186.09	9.53
Utilities	49	16.45	7.33	15.09	6.75	71.18	12.66
Wholesale (Non-dur)	51	8.36	8.70	7.91	8.27	22.90	25.09
Retail (Misc)	53	7.00	18.47	7.00	18.47	69.88	26.49
Banks	60	6.18	0.81	3.91	0.52	78.20	5.49
Insurance	63	7.27	3.62	6.91	3.46	33.77	4.11
Patent & Royalty	67	17.45	2.05	17.00	2.00	294.73	79.14
Hotels	70	2.45	7.71	2.45	7.71	2.00	5.00
Equipment Services	73	15.18	1.73	14.00	1.60	85.58	7.70
Health	80	11.18	10.17	11.18	10.17	54.73	67.06
Total	-	236.64	6.54	211.00	6.13	2043.84	23.21

Table II

Corporate Bond Summary Statistics

Summary statistics on the corporate bonds of the 235 sample firms that had some debt in their capital structure over 1994-2004. The third column reports the average number of bond issues per sample firm. Column four and five show the mean and standard deviation of the percentage of total book debt per firm-year that is observed through the FISD (%observed). %traded is the percentage of months that corporate bond issues are traded, weighted by bond issue face value. Column six and seven show the average and standard deviation of %traded across firms in the industry. The final row reports the average statistics across all industries. Source: FISD, Compustat.

Industry	2-digit	bond issues	%observed		%traded	
	SIC	per firm	mean	stdev	mean	stdev
Oil & Gas	13	10.55	73.45	25.03	65.26	12.31
Builders	15	6.70	61.07	32.85	71.45	6.12
Food	20	8.10	62.14	27.24	79.58	13.50
Paper	26	7.89	55.23	30.59	73.26	11.23
Publishing	27	2.67	62.17	24.11	70.90	10.25
Chemicals	28	5.82	63.53	29.83	72.60	13.11
Petroleum Products	29	6.58	45.99	23.36	74.48	10.54
Primary Metals	33	2.63	78.21	23.64	69.48	20.33
Machinery	35	3.86	64.03	31.51	63.79	11.38
Electric Equipment	36	3.22	80.39	26.03	70.45	11.49
Cars	37	6.56	48.03	30.45	76.02	12.37
Instruments	38	3.84	78.96	23.91	70.90	16.37
Transport (Air)	45	6.33	64.44	27.26	67.04	10.93
Telecom	48	7.44	76.72	30.41	71.06	14.10
Utilities	49	27.44	32.41	22.68	73.14	14.68
Wholesale (Non-dur)	51	3.90	65.16	29.23	66.18	15.88
Retail (Misc)	53	23.45	62.75	34.29	68.80	15.08
Banks	60	3.49	8.10	8.74	79.74	13.87
Insurance	63	2.23	63.73	24.77	65.06	8.76
Patent & Royalty	67	6.85	28.02	35.46	76.26	9.29
Hotels	70	3.99	93.72	13.54	71.23	13.70
Equipment Services	73	4.83	76.26	30.52	70.06	18.69
Health	80	3.43	65.80	32.02	67.12	18.04
Overall average	-	7.03	61.32	26.85	71.04	13.13

Table III

Spread in Financial Leverage

The spread in financial leverage over all firm-years in the sample is measured using: i) the ratio of the market value of debt (net of cash) divided by the sum of market value of debt (net of cash) and equity (D/A), and; ii) interest cover, defined as EBITDA divided by interest expense. The average and standard deviation across firm-years are reported for both measures. The final two columns report the range of credit ratings observed in each industry over the sample period. Source: FISD, CRSP, Compustat.

Industry	2-digit	D/A (market)		Interest Cover		Credit rating	
	SIC	mean	stdev	mean	stdev	min	max
Oil & Gas	13	0.25	0.18	10.91	5.96	B	A
Builders	15	0.45	0.17	6.01	6.88	B+	BBB
Food	20	0.23	0.15	10.24	5.88	BB+	AAA
Paper	26	0.46	0.08	4.73	2.18	B	A
Publishing	27	0.18	0.11	14.85	6.07	BBB-	AA
Chemicals	28	0.18	0.13	15.13	5.94	B	AAA
Petroleum Products	29	0.29	0.14	9.33	5.34	BB	A-
Primary Metals	33	0.31	0.23	6.96	3.77	B	A+
Machinery	35	0.27	0.23	8.96	5.81	B-	AA
Electric Equipment	36	0.20	0.14	11.19	5.51	B	AA+
Cars	37	0.28	0.18	10.27	6.23	B+	A+
Instruments	38	0.16	0.11	14.71	5.32	BB	AA-
Transport (Air)	45	0.38	0.25	7.00	5.43	D	A
Telecom	48	0.38	0.24	4.64	5.51	D	AA
Utilities	49	0.46	0.12	6.04	5.27	B	AA
Wholesale (Non-dur)	51	0.43	0.18	6.61	5.98	B	A
Retail (Misc)	53	0.31	0.19	8.20	6.10	BB-	A+
Banks	60	0.33	0.22	17.66	5.73	BBB	A+
Insurance	63	0.18	0.12	15.34	4.07	BB+	AA
Patent & Royalty	67	0.48	0.21	8.06	5.90	BB-	AAA
Hotels	70	0.54	0.00	3.92	1.52	BB-	BBB-
Equipment Services	73	0.19	0.20	13.71	5.71	D	AAA
Health	80	0.27	0.18	8.82	5.68	B	A+
Overall average	-	0.31	0.16	9.71	5.30	D	AAA

Table IV

Parameter Estimates
Unobserved Debt at Face Value

This table reports the posterior mean and standard deviation of parameter estimates over the 1994-2004 sample of 269 U.S. firms' monthly debt and equity values. The book value of the unobserved portion of debt is used as a proxy for its market value. The model parameters are estimated separately for each industry, using the MCMC algorithm described in appendix B. The net benefit of debt financing as a fraction of firm value, B/V^L, is a quadratic function of leverage, L: $B_t/V_t^L = \theta_0 + \theta_1 L_t + \theta_2 L_t^2$. Leverage is defined as the market value of debt divided by the market value of the firm (debt + equity). Debt is calculated net of cash.

Industry	2-digit	θ_0		θ_1		θ_2		MSE
	SIC	mean	s.d.	mean	s.d.	mean	s.d.	
Oil & Gas	13	0.000	0.000	0.295	0.006	-0.483	0.037	0.049
Builders	15	0.000	0.000	0.371	0.032	-0.376	0.050	0.037
Food	20	0.000	0.000	0.493	0.007	-1.154	0.116	0.062
Paper	26	0.047	0.002	0.358	0.064	-0.408	0.030	0.053
Publishing	27	0.000	0.000	0.690	0.009	-1.563	0.103	0.093
Chemicals	28	0.000	0.000	0.473	0.045	-1.898	0.311	0.031
Petroleum Products	29	0.049	0.000	0.240	0.045	-0.522	0.098	0.051
Primary Metals	33	0.033	0.014	0.078	0.051	-0.208	0.041	0.017
Machinery	35	0.000	0.000	0.397	0.005	-0.745	0.080	0.248
Electric Equipment	36	0.000	0.000	0.495	0.004	-1.633	0.170	0.051
Cars	37	0.000	0.000	0.359	0.045	-0.709	0.099	0.030
Instruments	38	0.000	0.000	0.382	0.012	-0.721	0.142	0.113
Transport (Air)	45	0.049	0.001	0.333	0.060	-0.560	0.073	0.010
Telecom	48	0.000	0.000	0.297	0.043	-0.482	0.044	0.100
Utilities	49	0.000	0.000	0.398	0.002	-0.282	0.046	0.036
Wholesale (Non-dur)	51	0.000	0.000	0.398	0.002	-0.589	0.059	0.014
Retail (Misc)	53	0.049	0.001	0.387	0.013	-0.752	0.072	0.025
Banks	60	0.000	0.000	0.493	0.008	-0.272	0.022	0.062
Insurance	63	0.000	0.000	0.376	0.020	-0.493	0.249	0.304
Patent & Royalty	67	0.000	0.000	0.292	0.061	-0.274	0.034	0.018
Hotels	70	0.000	0.000	0.307	0.035	-0.334	0.080	0.004
Equipment Services	73	0.000	0.000	0.329	0.050	-0.793	0.074	0.075
Health	80	0.033	0.015	0.194	0.018	-0.464	0.030	0.020
Average	-	0.011	0.002	0.367	0.028	-0.683	0.090	0.065

Table V

Parameter Estimates
Unobserved Debt at Credit Spread of Safest Bonds

This table reports the posterior mean and standard deviation of parameter estimates over the 1994-2004 sample of 269 U.S. firms' monthly debt and equity values. The market value of the unobserved portion of debt is calculated using the credit spread of the safest group of bonds. The model parameters are estimated separately for each industry, using the MCMC algorithm described in appendix B. The net benefit of debt financing as a fraction of firm value, B/V^L, is a quadratic function of leverage, L: $B_t/V_t^L = \theta_0 + \theta_1 L_t + \theta_2 L_t^2$. Leverage is defined as the market value of debt divided by the market value of the firm (debt + equity). Debt is calculated net of cash.

Industry	2-digit	θ_0		θ_1		θ_2		MSE
	SIC	mean	s.d.	mean	s.d.	mean	s.d.	
Oil & Gas	13	0.000	0.000	0.297	0.003	-0.518	0.023	0.043
Builders	15	0.000	0.000	0.322	0.067	-0.426	0.084	0.043
Food	20	0.000	0.001	0.404	0.033	-1.029	0.053	0.039
Paper	26	0.049	0.001	0.388	0.019	-0.531	0.020	0.002
Publishing	27	0.000	0.000	0.693	0.008	-1.655	0.185	0.057
Chemicals	28	0.000	0.000	0.477	0.019	-1.843	0.288	0.024
Petroleum Products	29	0.047	0.003	0.252	0.047	-0.597	0.068	0.026
Primary Metals	33	0.030	0.007	0.116	0.017	-0.302	0.015	0.017
Machinery	35	0.000	0.000	0.394	0.016	-0.823	0.041	0.027
Electric Equipment	36	0.000	0.000	0.494	0.005	-1.546	0.172	0.050
Cars	37	0.000	0.000	0.382	0.014	-0.734	0.086	0.053
Instruments	38	0.000	0.000	0.354	0.040	-0.680	0.096	0.031
Transport (Air)	45	0.046	0.002	0.125	0.012	-0.303	0.010	0.017
Telecom	48	0.001	0.000	0.398	0.006	-0.573	0.032	0.056
Utilities	49	0.000	0.000	0.333	0.053	-0.432	0.100	0.033
Wholesale (Non-dur)	51	0.000	0.000	0.395	0.012	-0.708	0.021	0.010
Retail (Misc)	53	0.049	0.001	0.396	0.001	-0.775	0.017	0.049
Banks	60	-0.000	0.000	0.404	0.072	-0.245	0.058	0.068
Insurance	63	0.000	0.000	0.341	0.043	-0.799	0.133	0.018
Patent & Royalty	67	0.005	0.006	0.162	0.043	-0.213	0.056	0.005
Hotels	70	0.049	0.001	0.059	0.015	-0.268	0.020	0.007
Equipment Services	73	0.001	0.005	0.350	0.070	-0.758	0.212	0.045
Health	80	0.041	0.011	0.224	0.047	-0.505	0.081	0.029
Average	-	0.014	0.002	0.337	0.029	-0.707	0.082	0.033

Table VI

Upper Bound on Costs of Financial Distress Unobserved Debt at Face Value

This table reports the upper bound on (ex-ante) expected costs of financial distress as a fraction of levered firm value (CFD), for different leverage ratios, L. The final column shows the upper bound on CFD at the observed industry leverage (aggregate book debt over all industry constituents in Compustat divided by aggregate book debt plus market value of equity, with aggregate debt net of cash), averaged over the sample period 1994-2004. The estimated model is $B_t/V_t^L = \theta_0 + \theta_1 L_t + \theta_2 L_t^2$. The upper bound on CFD is $-\theta_2 L_t^2$. The parameters are estimated separately for each industry, using the book value of the unobserved portion of debt as a proxy for its market value, as reported in table IV. The results are calculated using the posterior mean of θ_2.

Industry	2-digit SIC	L = 0.1	0.3	0.5	0.7	0.9	Observed L
Oil & Gas	13	0.005	0.043	0.121	0.237	0.391	0.030
Builders	15	0.004	0.034	0.094	0.184	0.305	0.076
Food	20	0.012	0.104	0.288	0.565	0.934	0.032
Paper	26	0.004	0.037	0.102	0.200	0.330	0.038
Publishing	27	0.016	0.141	0.391	0.766	1.266	0.064
Chemicals	28	0.019	0.171	0.475	0.930	1.538	0.021
Petroleum Products	29	0.005	0.047	0.131	0.256	0.423	0.008
Primary Metals	33	0.002	0.019	0.052	0.102	0.168	0.018
Machinery	35	0.007	0.067	0.186	0.365	0.603	0.055
Electric Equipment	36	0.016	0.147	0.408	0.800	1.323	0.027
Cars	37	0.007	0.064	0.177	0.347	0.574	0.163
Instruments	38	0.007	0.065	0.180	0.353	0.584	0.017
Transport (Air)	45	0.006	0.050	0.140	0.275	0.454	0.129
Telecom	48	0.005	0.043	0.120	0.236	0.390	0.030
Utilities	49	0.003	0.025	0.070	0.138	0.228	0.058
Wholesale (Non-dur)	51	0.006	0.053	0.147	0.289	0.477	0.027
Retail (Misc)	53	0.008	0.068	0.188	0.368	0.609	0.031
Banks	60	0.003	0.024	0.068	0.133	0.220	0.073
Insurance	63	0.005	0.044	0.123	0.242	0.399	0.057
Patent & Royalty	67	0.003	0.025	0.068	0.134	0.222	0.064
Hotels	70	0.003	0.030	0.083	0.164	0.270	0.034
Equipment Services	73	0.008	0.071	0.198	0.389	0.642	0.019
Health	80	0.005	0.042	0.116	0.227	0.376	0.038
Average	-	0.007	0.061	0.171	0.335	0.553	0.048

Table VII

Lower Bound on Costs of Financial Distress
Unobserved Debt at Face Value

This table reports the lower bound on (ex-ante) expected costs of financial distress as a fraction of levered firm value (CFD) for different leverage ratios, L. The final column shows the lower bound on CFD at the observed industry leverage (aggregate book debt over all industry constituents in Compustat divided by aggregate book debt plus market value of equity, with aggregate debt net of cash), averaged over the sample period 1994-2004. The estimated model is $B_t/V_t^L = \theta_0 + \theta_1 L_t + \theta_2 L_t^2$. The lower bound on CFD is $\max(0, -\theta_1 L_t - \theta_2 L_t^2)$. The parameters are estimated separately for each industry, using the book value of the unobserved portion of debt as a proxy for its market value, as reported in table IV. The results are calculated using the posterior mean of θ_1 and θ_2.

Industry	2-digit			L =			Observed
	SIC	0.1	0.3	0.5	0.7	0.9	L
Oil & Gas	13	0.000	0.000	0.000	0.030	0.125	0.000
Builders	15	0.000	0.000	0.000	0.000	0.000	0.000
Food	20	0.000	0.000	0.042	0.220	0.491	0.000
Paper	26	0.000	0.000	0.000	0.000	0.008	0.000
Publishing	27	0.000	0.000	0.046	0.283	0.645	0.000
Chemicals	28	0.000	0.029	0.238	0.599	1.112	0.000
Petroleum Products	29	0.000	0.000	0.011	0.088	0.207	0.000
Primary Metals	33	0.000	0.000	0.013	0.048	0.098	0.000
Machinery	35	0.000	0.000	0.000	0.087	0.247	0.000
Electric Equipment	36	0.000	0.000	0.161	0.454	0.877	0.000
Cars	37	0.000	0.000	0.000	0.096	0.251	0.000
Instruments	38	0.000	0.000	0.000	0.086	0.240	0.000
Transport (Air)	45	0.000	0.000	0.000	0.041	0.154	0.000
Telecom	48	0.000	0.000	0.000	0.028	0.123	0.000
Utilities	49	0.000	0.000	0.000	0.000	0.000	0.000
Wholesale (Non-dur)	51	0.000	0.000	0.000	0.010	0.119	0.000
Retail (Misc)	53	0.000	0.000	0.000	0.097	0.261	0.000
Banks	60	0.000	0.000	0.000	0.000	0.000	0.000
Insurance	63	0.000	0.000	0.000	0.000	0.061	0.000
Patent & Royalty	67	0.000	0.000	0.000	0.000	0.000	0.000
Hotels	70	0.000	0.000	0.000	0.000	0.000	0.000
Equipment Services	73	0.000	0.000	0.034	0.158	0.346	0.000
Health	80	0.000	0.000	0.019	0.092	0.201	0.000
Average	-	0.000	0.001	0.024	0.105	0.242	0.000

Table VIII

Upper Bound on Costs of Financial Distress
Unobserved Debt at Credit Spread of Safest Bonds

This table reports the upper bound on (ex-ante) expected costs of financial distress as a fraction of levered firm value (CFD), for different leverage ratios, L. The final column shows the upper bound on CFD at the observed industry leverage (aggregate book debt over all industry constituents in Compustat divided by aggregate book debt plus market value of equity, with aggregate debt net of cash), averaged over the sample period 1994-2004. The estimated model is $B_t/V_t^L = \theta_0 + \theta_1 L_t + \theta_2 L_t^2$. The upper bound on CFD is $-\theta_2 L_t^2$. The parameters are estimated separately for each industry, using the credit spread of the safest group of bonds to calculate the market value of the unobserved portion of debt, as reported in table V. The results are calculated using the posterior mean of θ_2.

Industry	2-digit SIC	L = 0.1	0.3	0.5	0.7	0.9	Observed L
Oil & Gas	13	0.005	0.047	0.130	0.254	0.420	0.033
Builders	15	0.004	0.038	0.107	0.209	0.345	0.086
Food	20	0.010	0.093	0.257	0.504	0.833	0.028
Paper	26	0.005	0.048	0.133	0.260	0.430	0.050
Publishing	27	0.017	0.149	0.414	0.811	1.341	0.068
Chemicals	28	0.018	0.166	0.461	0.903	1.493	0.020
Petroleum Products	29	0.006	0.054	0.149	0.293	0.484	0.009
Primary Metals	33	0.003	0.027	0.076	0.148	0.245	0.026
Machinery	35	0.008	0.074	0.206	0.403	0.666	0.061
Electric Equipment	36	0.015	0.139	0.386	0.757	1.252	0.025
Cars	37	0.007	0.066	0.184	0.360	0.595	0.169
Instruments	38	0.007	0.061	0.170	0.333	0.551	0.016
Transport (Air)	45	0.003	0.027	0.076	0.148	0.245	0.070
Telecom	48	0.006	0.052	0.143	0.281	0.464	0.036
Utilities	49	0.004	0.039	0.108	0.212	0.350	0.088
Wholesale (Non-dur)	51	0.007	0.064	0.177	0.347	0.574	0.033
Retail (Misc)	53	0.008	0.070	0.194	0.380	0.628	0.032
Banks	60	0.002	0.022	0.061	0.120	0.199	0.066
Insurance	63	0.008	0.072	0.200	0.391	0.647	0.093
Patent & Royalty	67	0.002	0.019	0.053	0.104	0.172	0.049
Hotels	70	0.003	0.024	0.067	0.131	0.217	0.027
Equipment Services	73	0.008	0.068	0.190	0.371	0.614	0.018
Health	80	0.005	0.045	0.126	0.247	0.409	0.042
Average	-	0.007	0.064	0.177	0.346	0.573	0.050

Table IX

Lower Bound on Costs of Financial Distress
Unobserved Debt at Credit Spread of Safest Bonds

This table reports the lower bound on (ex-ante) expected costs of financial distress as a fraction of levered firm value (CFD), for different leverage ratios, L. The final column shows the lower bound on CFD at the observed industry leverage (aggregate book debt over all industry constituents in Compustat divided by aggregate book debt plus market value of equity, with aggregate debt net of cash), averaged over the sample period 1994-2004. The estimated model is $B_t/V_t^L = \theta_0 + \theta_1 L_t + \theta_2 L_t^2$. The lower bound on CFD is $\max(0, -\theta_1 L_t - \theta_2 L_t^2)$. The parameters are estimated separately for each industry, using the credit spread of the safest group of bonds to calculate the market value of the unobserved portion of debt, as reported in table V. The results are calculated using the posterior mean of θ_1 and θ_2.

Industry	2-digit SIC	L = 0.1	L = 0.3	L = 0.5	L = 0.7	L = 0.9	Observed L
Oil & Gas	13	0.000	0.000	0.000	0.046	0.153	0.000
Builders	15	0.000	0.000	0.000	0.000	0.055	0.000
Food	20	0.000	0.000	0.055	0.221	0.470	0.000
Paper	26	0.000	0.000	0.000	0.000	0.081	0.000
Publishing	27	0.000	0.000	0.068	0.326	0.717	0.000
Chemicals	28	0.000	0.023	0.222	0.569	1.064	0.000
Petroleum Products	29	0.000	0.000	0.023	0.116	0.256	0.000
Primary Metals	33	0.000	0.000	0.018	0.067	0.140	0.000
Machinery	35	0.000	0.000	0.009	0.128	0.312	0.000
Electric Equipment	36	0.000	0.000	0.140	0.412	0.808	0.000
Cars	37	0.000	0.000	0.000	0.092	0.251	0.000
Instruments	38	0.000	0.000	0.000	0.085	0.232	0.000
Transport (Air)	45	0.000	0.000	0.013	0.061	0.132	0.010
Telecom	48	0.000	0.000	0.000	0.002	0.106	0.000
Utilities	49	0.000	0.000	0.000	0.000	0.050	0.000
Wholesale (Non-dur)	51	0.000	0.000	0.000	0.070	0.218	0.000
Retail (Misc)	53	0.000	0.000	0.000	0.102	0.271	0.000
Banks	60	0.000	0.000	0.000	0.000	0.000	0.000
Insurance	63	0.000	0.000	0.029	0.153	0.341	0.000
Patent & Royalty	67	0.000	0.000	0.000	0.000	0.027	0.000
Hotels	70	0.000	0.006	0.037	0.090	0.164	0.009
Equipment Services	73	0.000	0.000	0.015	0.127	0.299	0.000
Health	80	0.000	0.000	0.014	0.091	0.207	0.000
Average	-	0.000	0.001	0.028	0.120	0.276	0.001

Table X

Estimates of Ex-post Costs of Financial Distress

This table shows the ex-post costs of financial distress as a fraction of firm value. Ex-post costs of financial distress ("loss-given-default") are the costs of financial distress once bankruptcy has been filed, and equal $-(\theta_1+\theta_2)$, as a fraction of firm value. The third and fourth columns show the posterior mean and standard deviation of ex-post CFD when parameters are estimated using the book value of the unobserved debt as a proxy for its market value, as reported in table IV. The fifth and sixth columns show the mean and standard deviation of ex-post CFD when parameters are estimated using the credit spread of the safest group of bonds to calculate the market value of the unobserved debt, as reported in table V.

Industry	2-digit	Face		Credit spread	
	SIC	mean	s.d.	mean	s.d.
Oil & Gas	13	0.188	0.038	0.221	0.023
Builders	15	0.005	0.019	0.104	0.020
Food	20	0.661	0.113	0.625	0.022
Paper	26	0.050	0.068	0.143	0.017
Publishing	27	0.873	0.103	0.963	0.185
Chemicals	28	1.425	0.269	1.366	0.281
Petroleum Products	29	0.282	0.119	0.345	0.023
Primary Metals	33	0.130	0.013	0.186	0.005
Machinery	35	0.348	0.080	0.429	0.030
Electric Equipment	36	1.138	0.170	1.052	0.172
Cars	37	0.350	0.063	0.352	0.081
Instruments	38	0.339	0.143	0.326	0.082
Transport (Air)	45	0.227	0.014	0.177	0.005
Telecom	48	0.185	0.005	0.175	0.033
Utilities	49	-0.116	0.046	0.099	0.047
Wholesale (Non-dur)	51	0.191	0.059	0.313	0.010
Retail (Misc)	53	0.365	0.069	0.378	0.017
Banks	60	-0.221	0.025	-0.159	0.061
Insurance	63	0.117	0.241	0.458	0.093
Patent & Royalty	67	-0.018	0.048	0.051	0.042
Hotels	70	0.027	0.071	0.209	0.006
Equipment Services	73	0.464	0.031	0.408	0.204
Health	80	0.270	0.030	0.281	0.037
Average	-	0.262	0.306	0.314	0.256

Table XI

Regressions of Distress Parameter θ_2

OLS regression results of estimated posterior mean θ_2 (from table IV) on industry characteristics. All explanatory variables are at the industry level (value-weighted and averaged over the sample period 1994-2004, except Log(Assets) which is equally weighted), calculated over all industry constituents in Compustat. Intang/Assets is the level of intangibles relative to book assets. EBITDA / Sales is the annual earnings before interest, taxes, depreciation and amortization relative to total sales. R&D / Sales is the annual R&D expense relative to total sales. M/B is the market-to-book ratio. Wages / Sales is annual labor expense relative to total sales. Machinery is an industry dummy that equals one for SIC codes 35-39 and zero otherwise. Services is an industry dummy that equals one for SIC codes 73 and 80 and zero otherwise. Liquidity is the average monthly number of shares traded relative to shares outstanding. Log(Assets) is the natural logarithm of book assets. Standard errors are in parentheses. ***, **, and * denote significance at the 1%, 5% and 10% levels, respectively.

Specification:	I	II	III	IV	V	VI	VII
Intang /Assets	-2.343	-2.712	.	.	.	.	.
	(0.946)**	(1.068)**	.	.	.	.	.
Intang /Assets	.	2.308	.	.	.	.	.
* EBITDA / Sales	.	(2.990)	.	.	.	.	.
R&D / Sales	.	.	-7.209	.	.	.	.
	.	.	(1.210)***	.	.	.	.
M/B	.	.	.	-0.208	.	.	.
	.	.	.	(0.050)***	.	.	.
Wages / Sales	.	.	.	.	-1.421	.	.
	.	.	.	.	(0.830)	.	.
Machinery = 1	.	.	.	.	.	-0.326	.
	.	.	.	.	.	(0.257)	.
Services = 1	.	.	.	.	.	-0.002	.
	.	.	.	.	.	(0.346)	.
Liquidity	.	.	.	.	.	.	0.486
	.	.	.	.	.	.	(2.616)
Log (Assets)	.	.	.	.	.	.	0.081
	.	.	.	.	.	.	(0.091)
Intercept	-0.424	-0.401	-0.270	0.084	-0.380	-0.626	-0.779
	(0.136)***	(0.130)***	(0.092)***	(0.196)	(0.200)*	(0.112)***	(0.358)**
adjusted R^2	0.189	0.173	0.611	0.431	0.081	-0.017	-0.059
F	6.133	3.306	35.525	17.645	2.932	0.818	0.417
p	0.022	0.058	0.000	0.000	0.102	0.456	0.665
N	23	23	23	23	23	23	23

Table XII

Regressions of Observed Leverage on Distress Parameters

OLS regressions results of observed leverage on distress parameters and industry characteristics. The dependent variable is observed industry leverage, calculated as the aggregate book debt over all industry constituents in Compustat divided by aggregate book debt plus market value of equity, net of cash and averaged over the sample period 1994-2004. θ_1 and θ_2 are the posterior mean estimates from table IV. θ_1/θ_2 is the posterior mean of the distribution of θ_1 divided by θ_2. σ^V is an industry measure of unlevered firm volatility, calculated as the posterior mean standard deviation of unlevered asset returns, averaged over the sample firms in each industry. The other explanatory variables are as defined in table XI. Standard errors are in parentheses. ***, **, and * denote significance at the 1%, 5% and 10% levels, respectively.

Specification:	I	II	III	IV	V	VI
θ_1	0.528	.	.	.	.	.
	(0.228)**	.	.	.	.	.
θ_2	0.270	.	.	.	.	.
	(0.060)***	.	.	.	.	.
θ_1/θ_2	.	-0.248	.	-0.151	.	-0.161
	.	(0.053)***	.	(0.063)**	.	(0.053)***
Intang / Assets	.	.	-0.302	-0.106	.	.
	.	.	(0.293)	(0.270)	.	.
R&D / Sales	.	.	-0.102	-0.275	.	.
	.	.	(0.628)	(0.557)	.	.
M/B	.	.	-0.051	-0.036	-0.063	-0.042
	.	.	(0.019)**	(0.018)*	(0.013)***	(0.013)***
EBITDA / Sales	.	.	0.085	0.060	.	.
	.	.	(0.067)	(0.059)	.	.
σ^V	.	.	-0.005	0.076	.	.
	.	.	(0.287)	(0.255)	.	.
Log(Assets)	.	.	-0.007	-0.012	.	.
	.	.	(0.019)	(0.017)	.	.
Intercept	0.279	0.118	0.532	0.369	0.522	0.332
	(0.067)***	(0.042)**	(0.081)***	(0.098)***	(0.053)***	(0.077)***
adjusted R^2	0.465	0.485	0.435	0.564	0.492	0.637
F	10.559	21.705	3.826	5.069	22.272	20.324
p	0.001	0.000	0.015	0.004	0.000	0.000
N	23	23	23	23	23	23

Table XIII

Gains and Costs to Adjusting Leverage

The gain in firm value from adjusting leverage (column 5) is calculated as the difference in total firm value at the tenth percentile of the industry's leverage distribution and firm value at the industry's optimal leverage ratio, using the parameters from table IV. Column 6 shows the cost of the leverage adjustment if direct issuance costs are 6.5% of issue size. Both the gain and cost of adjusting leverage are expressed as a fraction of firm value. Leverage (L) is measured as the book value of debt divided by the book value of debt plus market value of equity for each Compustat firm in the industry, for each sample year separately, and with debt net of cash. The standard deviation of L across firm-years and the (10%,90%) interval are shown in columns 3 and 4.

Industry	2-digit SIC	s.d.(L)	L (10%,90%)	Gain from relevering	Adj. cost
Oil & Gas	13	0.212	(0.052, 0.597)	0.014	0.018
Builders	15	0.237	(0.170, 0.816)	0.008	0.026
Food	20	0.215	(0.044, 0.595)	0.013	0.013
Paper	26	0.231	(0.105, 0.734)	0.019	0.024
Publishing	27	0.218	(0.017, 0.559)	0.030	0.014
Chemicals	28	0.198	(0.004, 0.447)	0.021	0.008
Petroleum Products	29	0.192	(0.082, 0.546)	0.005	0.011
Primary Metals	33	0.254	(0.096, 0.790)	0.000	0.009
Machinery	35	0.214	(0.004, 0.525)	0.041	0.017
Electric Equipment	36	0.211	(0.004, 0.500)	0.025	0.010
Cars	37	0.246	(0.040, 0.687)	0.010	0.015
Instruments	38	0.188	(0.003, 0.409)	0.045	0.017
Transport (Air)	45	0.283	(0.073, 0.862)	0.003	0.018
Telecom	48	0.262	(0.044, 0.754)	0.014	0.019
Utilities	49	0.176	(0.223, 0.656)	0.040	0.038
Wholesale (Non-dur)	51	0.253	(0.029, 0.697)	0.034	0.021
Retail (Misc)	53	0.272	(0.034, 0.758)	0.018	0.016
Banks	60	0.229	(0.133, 0.754)	0.104	0.055
Insurance	63	0.188	(0.046, 0.483)	0.040	0.023
Patent & Royalty	67	0.246	(0.086, 0.790)	0.016	0.034
Hotels	70	0.277	(0.148, 0.904)	0.012	0.025
Equipment Services	73	0.220	(0.001, 0.494)	0.031	0.013
Health	80	0.257	(0.016, 0.705)	0.007	0.013
Average	-	0.230	(0.063, 0.655)	0.024	0.013

Table XIV

Simulation Parameters

This table lists the model's parameters with a brief description of their meaning, and the value used in simulation. The PV of the net benefit of financing relative to total firm value, B/V^L, is a quadratic function of leverage, L: $B_t/V_t^L = \theta_0 + \theta_1 L_t + \theta_2 L_t^2 + u_t$. The u_t have mean zero and variance R. The relation between an individual firm's debt, equity and asset betas is subject to an error, v_t, with mean zero, variance S and correlation ρ with u_t. Both u_t and v_t are i.i.d. over time and independent across firms. The industry unlevered asset beta follows an AR(1) process $\beta_{t+1}^U = \phi_0 + \phi_1 \beta_t^U + \eta_t$. Debt and equity returns are generated from the CAPM, where the idiosyncratic returns are correlated with coefficient ρ^{DE}.

Parameter	Definition	Value
Costs of Financial Distress (CFD)		
θ_0	CFD when leverage = 0	0
θ_1	Marginal tax rate	0.2
θ_2	CFD parameter	-0.3
R	Variance of error u_t	0.005^2
S	Variance of error v_t	0.05^2
ρ	Correlation between u_t and v_t	0.1
Unlevered asset beta (β^V)		
β_0^U	Starting value	1
ϕ_0	Intercept of AR(1)	0.02
ϕ_1	Auto-regressive coefficient	0.98
H	Variance of AR(1) error η_t	0.01^2
Firm-specific returns		
σ^E	Annualized s.d. of idiosynractic shock to equity returns	0.1
σ^D	Annualized s.d. of idiosynractic shock to debt returns	0.05
ρ^{DE}	Correlation between idiosyncratic shocks to debt and equity	0.3
Market parameters		
r_f	Risk-free rate (constant, annualized)	0.04
$E(r_t^M - r_f)$	Annualized expected excess market return	0.07
$\sigma(r_t^M - r_f)$	Annualized volatility of excess market return	0.15

Table XV

Parameter Estimates from Simulations

This table shows summary statistics of MCMC posterior means and standard deviations from 100 simulated datasets of equal length and frequency as the empirical data in the paper. The parameter values used for the simulations are shown in table XIV. The parameters θ_0, θ_1 and θ_2 are estimated on each dataset of simulated debt and equity values, using the MCMC procedure described in appendix B. Four different scenarios are used: A) The market value of corporate debt is perfectly observed; B) debt market value is observed in only one half of the months; C) Same as scenario B but when observed, the market value of debt is biased upwards as if the model were fitted using book values of bank debt instead of market values; D) Same as A but unlevered asset betas are different across firms within the same industry.

Panel A: Perfectly Observed Debt				
	Posterior mean		Posterior s.d.	
	mean	s.d.	mean	s.d.
θ_0	0.001	0.000	0.001	0.000
θ_1	0.197	0.010	0.021	0.011
θ_2	-0.285	0.020	0.038	0.018

Panel B: 50% Observed Debt				
	Posterior mean		Posterior s.d.	
	mean	s.d.	mean	s.d.
θ_0	0.001	0.000	0.001	0.000
θ_1	0.199	0.026	0.024	0.030
θ_2	-0.286	0.042	0.041	0.042

Panel C: Bank Debt at Book Value				
	Posterior mean		Posterior s.d.	
	mean	s.d.	mean	s.d.
θ_0	0.001	0.000	0.001	0.001
θ_1	0.194	0.087	0.028	0.050
θ_2	-0.245	0.128	0.043	0.056

Panel D: Unlevered Asset Betas				
	Posterior mean		Posterior s.d.	
	mean	s.d.	mean	s.d.
θ_0	0.001	0.000	0.001	0.001
θ_1	0.200	0.018	0.025	0.034
θ_2	-0.297	0.108	0.043	0.056

Figure 1

Betas as a Function of Leverage

This figure depicts the relation between leverage and the beta of a firm's debt, equity and assets, with and without tax benefits and/or costs of financial distress. Leverage is defined as the market value of debt, D, divided by the total market value of the firm, $V^L = D + E$, with E the market value of the company's equity. β^D is the beta of the company's debt and β^E is the beta of the firm's equity. The levered firm beta is defined as the weighted average of debt and equity betas: $\beta^L = \frac{D}{V^L}\beta^D + \frac{E}{V^L}\beta^E$. The unlevered asset beta is assumed equal to 1.

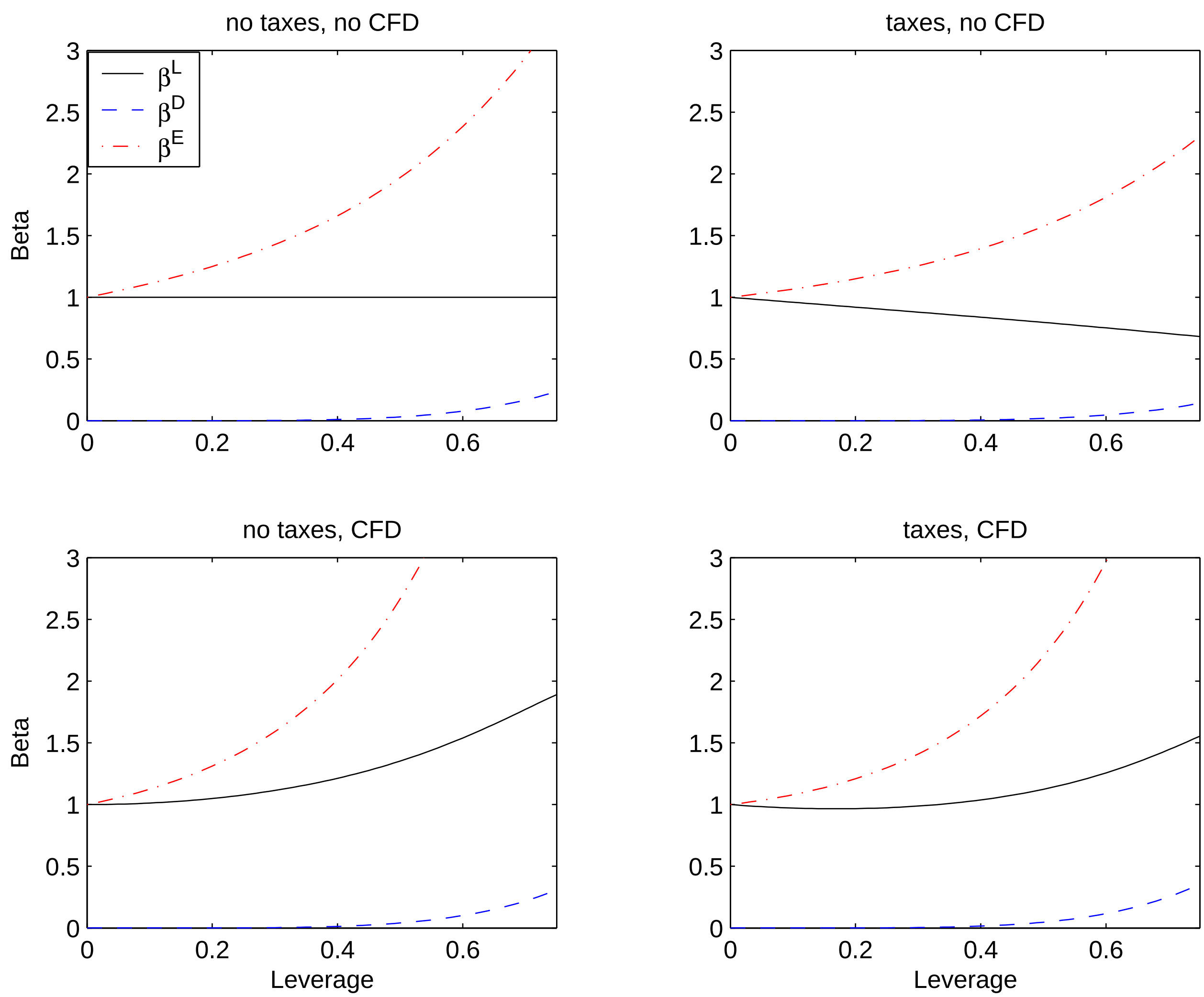

Figure 2

Levered Firm Beta as a Function of Leverage

This figure depicts the relation between financial leverage and the beta of the levered firm when there are tax benefits and costs of financial distress. The net benefit of financing is $B_t/V_t^L = 0.2L_t + \theta_2 L_t^2$, so that the marginal tax rate that determines the tax benefits of debt is 20%. Costs of financial distress are driven by $\theta_2 \leq 0$. When θ_2 equals zero, there are no CFD. The more negative θ_2, the higher the costs of financial distress at a given leverage ratio. The levered firm beta is defined as the weighted average of debt and equity betas: $\beta^L = \frac{D}{V^L}\beta^D + \frac{E}{V^L}\beta^E$. The unlevered beta is assumed equal to 1.

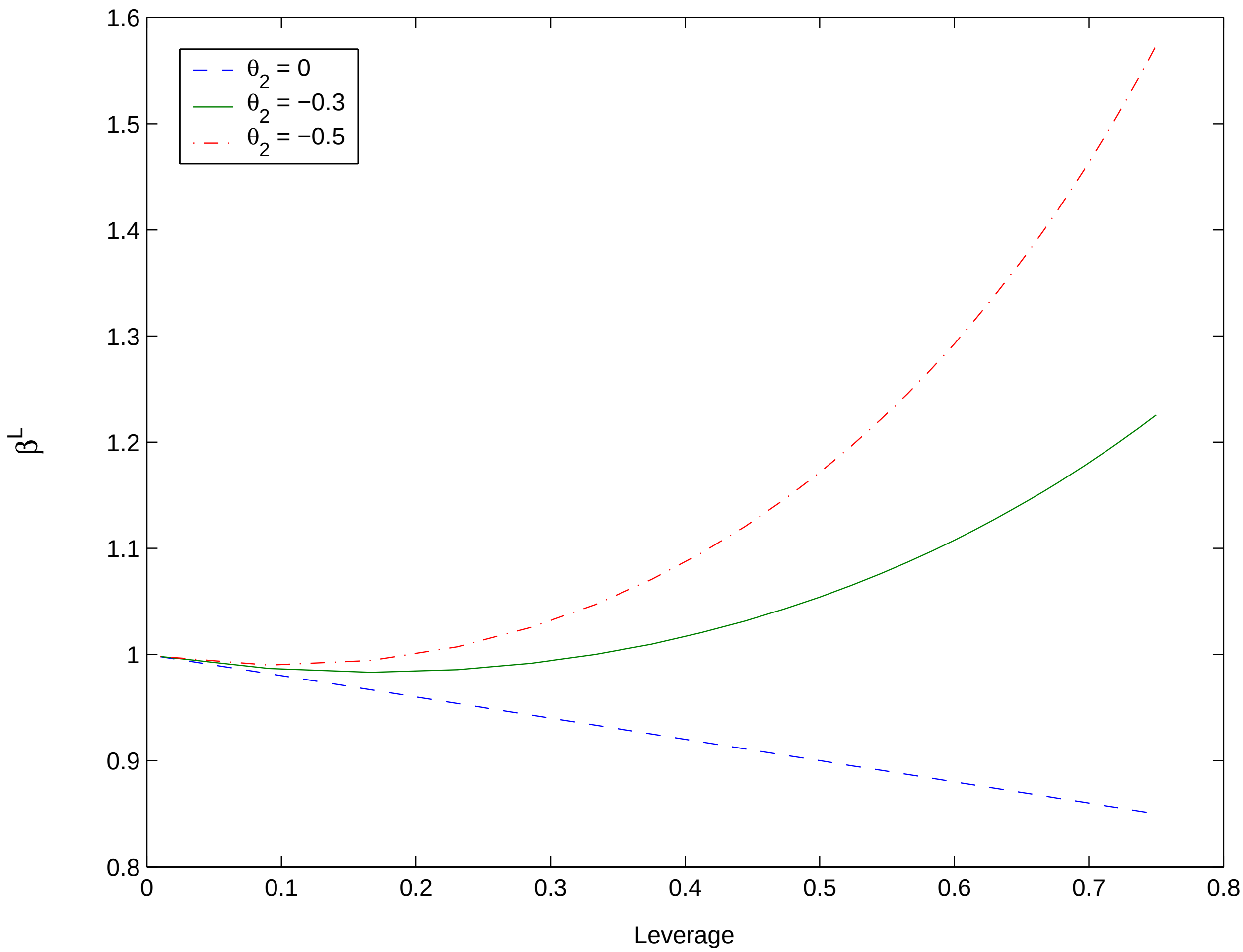

Figure 3

Model Fit: Oil & Gas Industry

The top plot shows the posterior mean of V^L/V^U versus leverage for each sample firm-month in the oil & gas industry (SIC code 13) and their estimated relationship, equation (15), using the posterior mean θ estimates. The bottom plot shows the posterior mean of β^L versus leverage. Since the unlevered asset beta of this industry changes substantially over time, the bottom panel shows the band of β^L's that is consistent with the range of β^U's over the sample period, from equation (16).

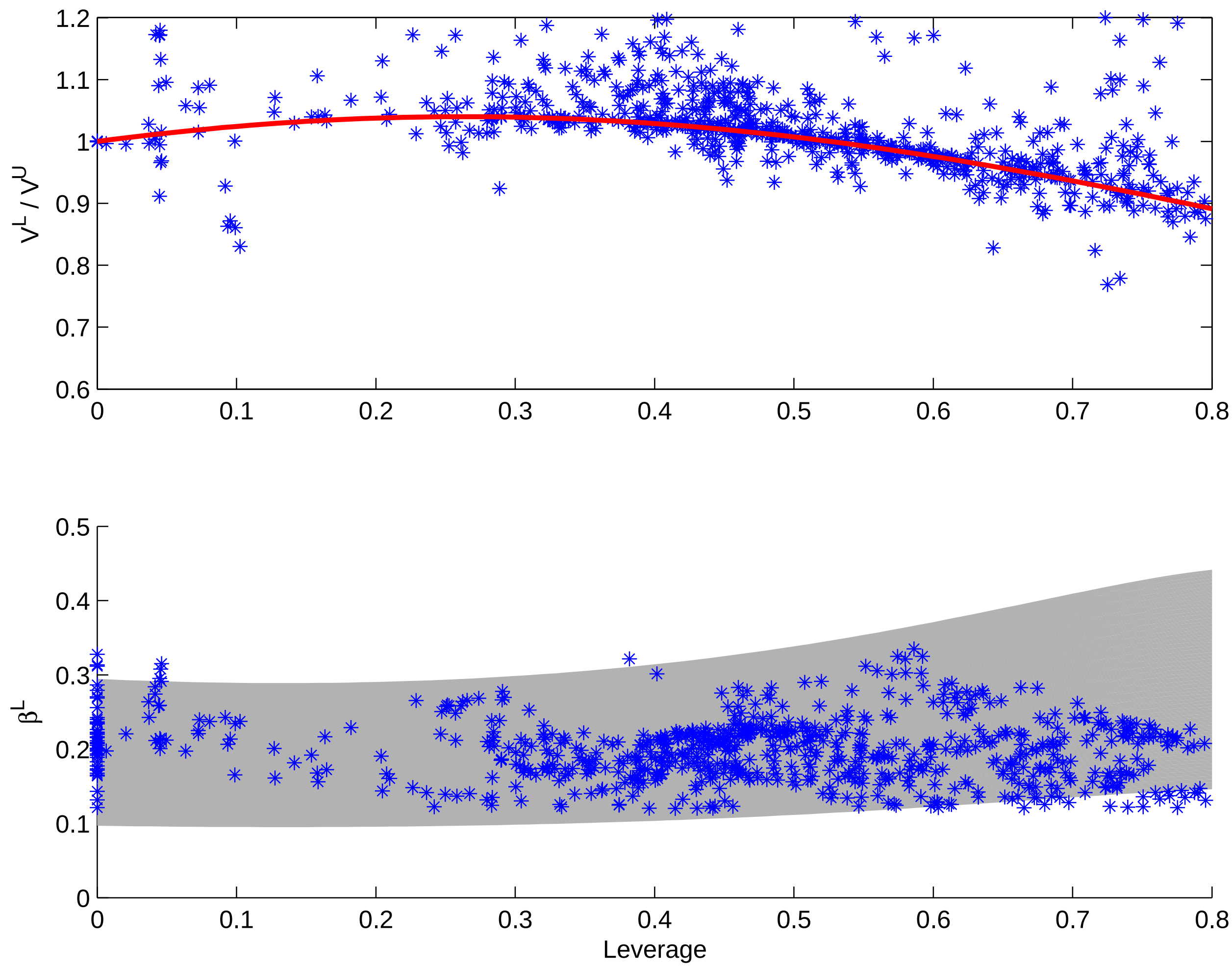

Figure 4

Unlevered Beta: Oil & Gas Industry

This figure shows the posterior mean of the estimated unlevered asset beta, β^U, of the oil & gas industry (SIC code 13) over the sample period 1994-2004.

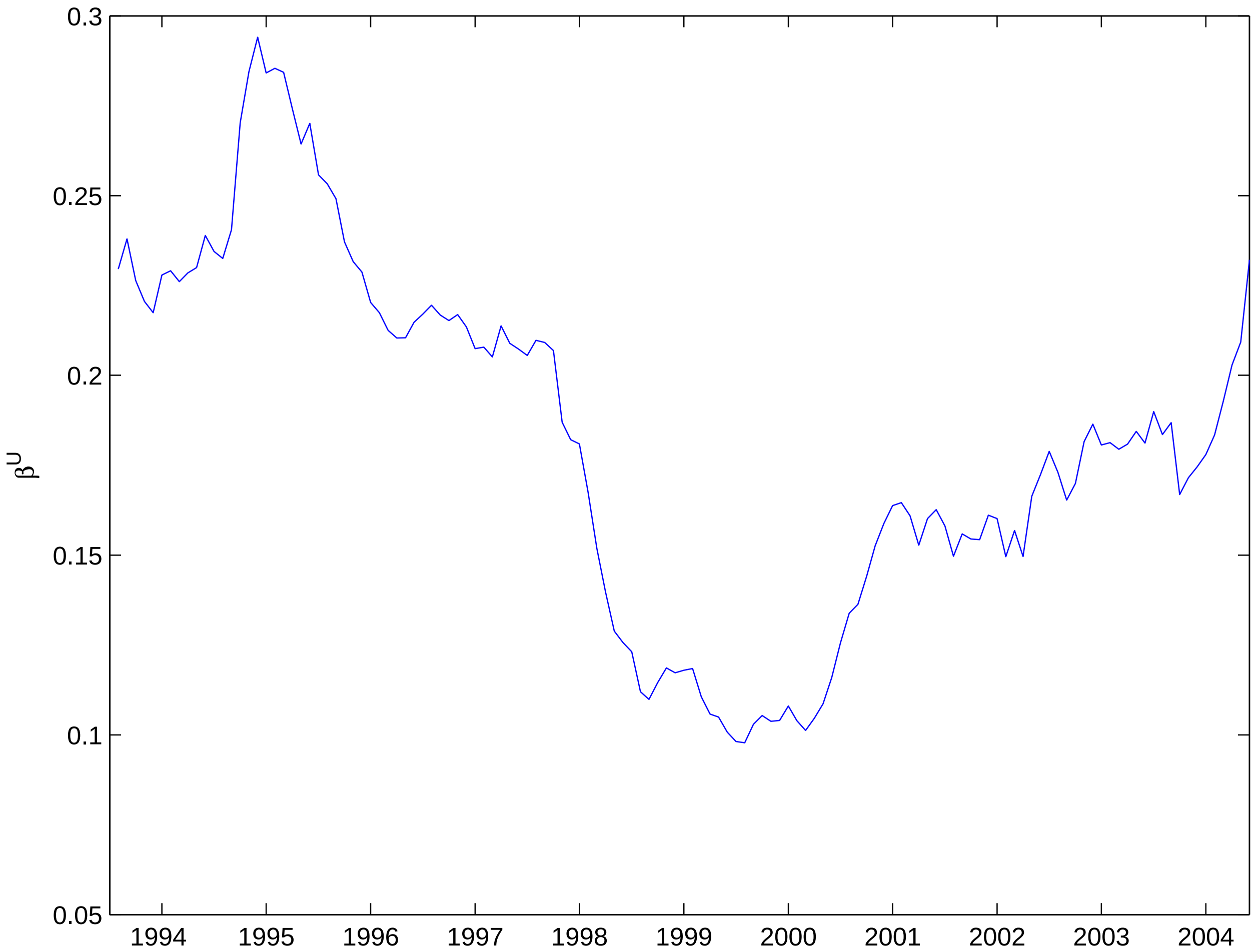

Figure 5

Time-series of Ex-ante Costs of Financial Distress

Time-series plot of the ex-ante costs of financial distress over 1963-2004, both equally-weighted and value-weighted across the sample industries. Costs of distress are calculated as in table VI, using industry leverage ratios (book value of debt over book value of debt plus market value of equity) from annual Compustat data of all firms in the sample industries. The shaded areas are peak-to-trough periods as defined by the NBER.

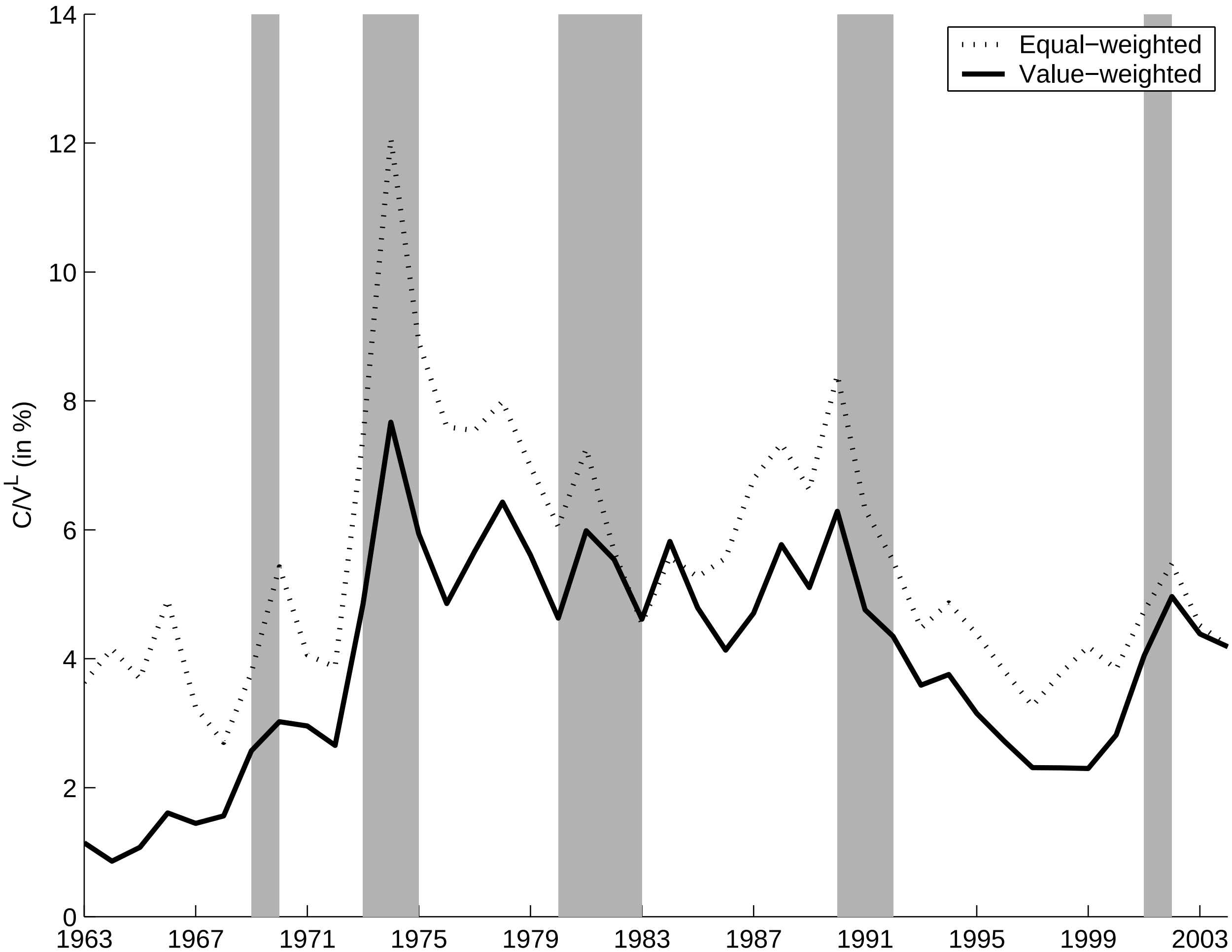

Figure 6

Implied Optimal Leverage versus Observed Leverage, Unobserved Debt at Face Value

Box-plot of the distribution of optimal leverage for each industry, as implied by the posterior distribution of model parameter estimates. The model is estimated using the market values of debt and equity for a subset of industry constituents, assuming the unobserved portion of debt trades at face value, as in table IV. Model-implied optimal leverage represents the optimal ratio of the market value of debt to the market value of debt plus equity and is calculated as $-\theta_1/2\theta_2$. The (red) horizontal line within the box is the median of the distribution, and the boundaries of the box represent the first and third quartiles. The horizontal lines outside the box are drawn at the minimum and maximum or 1.5 times the interquartile range, whichever is closer to the box. Values outside this range are considered outliers. The vertical lines extend to the minimum and maximum of the distribution. The (black) circles are the observed industry leverage ratios, calculated as the aggregate book debt, net of cash, over all industry constituents in Compustat divided by aggregate book debt plus market value of equity and averaged over the sample period 1994-2004. All debt data is net of cash.

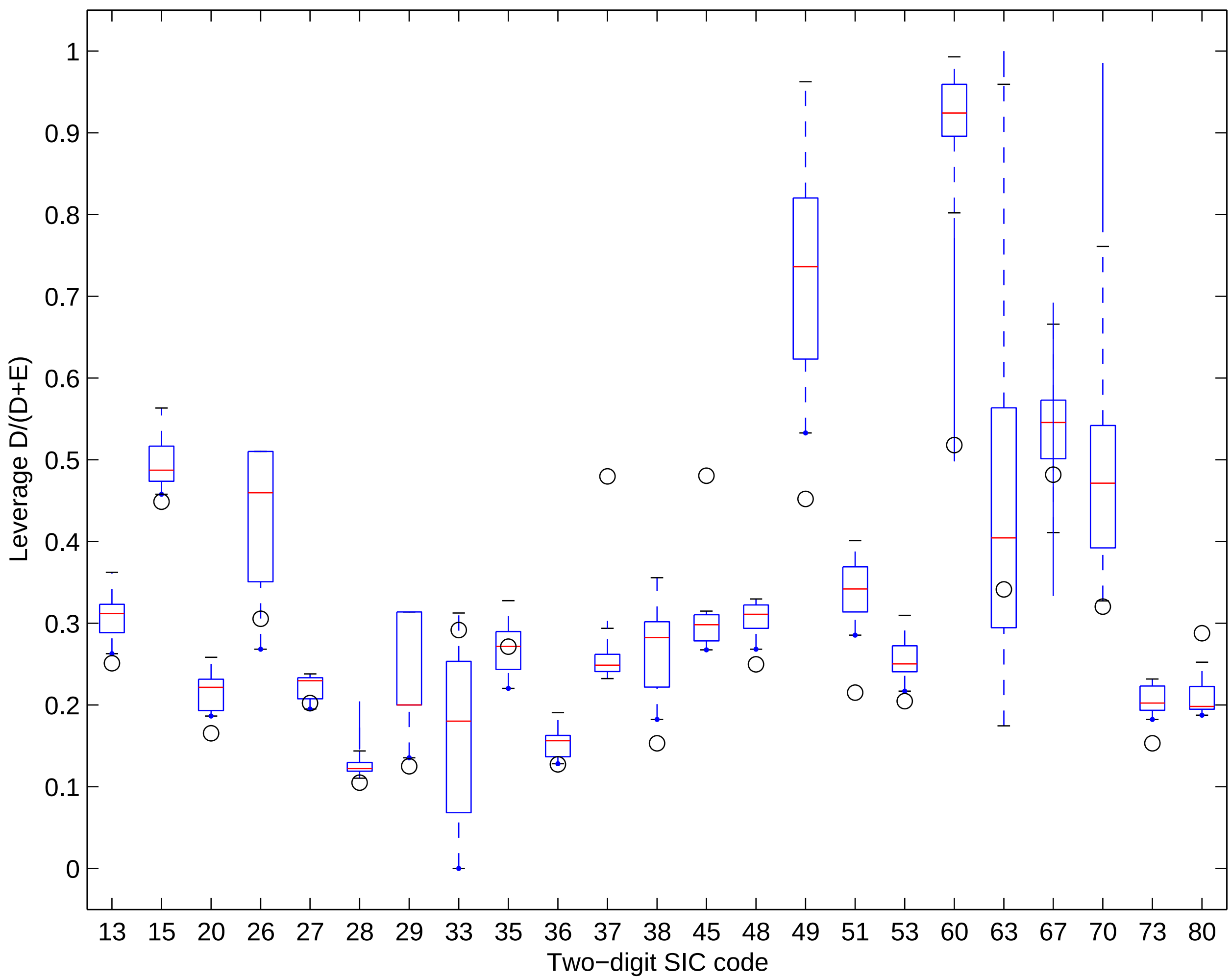

Figure 7

Implied Optimal Leverage versus Observed Leverage, Unobserved Debt at Credit Spread of Safest Bonds

Box-plot of the distribution of optimal leverage for each industry, as implied by the posterior distribution of model parameter estimates. The model is estimated using the market values of debt and equity for a subset of industry constituents, assuming the unobserved portion of debt is traded at the credit spread of the safest publicly-traded bond of the same firm, as in table V. Model-implied optimal leverage represents the optimal ratio of the market value of debt to the market value of debt plus equity and is calculated as $-\theta_1/2\theta_2$. The (red) horizontal line within the box is the median of the distribution, and the boundaries of the box represent the first and third quartiles. The horizontal lines outside the box are drawn at the minimum and maximum or 1.5 times the interquartile range, whichever is closer to the box. Values outside this range are considered outliers. The vertical lines extend to the minimum and maximum of the distribution. The (black) circles are the observed industry leverage ratios, calculated as the aggregate book debt over all industry constituents in Compustat divided by aggregate book debt plus market value of equity and averaged over the sample period 1994-2004. All debt data is net of cash.