Antenna Selection for Spatial Multiplexing Systems Based on Complex Householder QR Factorization

Zhibin Xie
Dept. of Information Science & Engineering
Northeastern University
Shenyang, China
xie_zhibin@126.com

Jinkuan Wang, Yun Wang
Dept. of Information Science & Engineering
Northeastern University
Shenyang, China
wjk@mail.neuq.edu.cn

Abstract—**In this paper, a novel antenna selection algorithm based on complex Householder QR factorization is proposed in wireless multiple input multiple output (MIMO) systems. It avoids the shortcoming of previous methods which need a mass of search over all possible subsets， has lower computation complexity and better performance. Simulation results demonstrate that the proposed antenna selection algorithm achieves excellent performance with linear receivers over fading channels.**

Keywords-complex Householder QR factorization; vector symbol error rate (VSER); antenna selection

I. Introduction

In a rich scattering environment, multiple antennas at transmit and receive sides can significantly improve the spectral efficiency. It has been known that the capacity of the system scales linearly with the minimum number of transmit and receive antennas [1], [2], and this has spurred a great flurry of research in recent years. However, this comes at the expense of increased hardware and signal processing complexity. Antenna selection method can reduce the hardware costs of a multiple input multiple output (MIMO) system, and have gained lots of interest recently. This reduces the number of RF chains required by a MIMO system, while maintaining most of the advantages of using multiple antennas.

Antenna selection algorithms, which have been intensively studied in many literatures, optimize different performance metrics, such as, information theoretic capacity, probability of error, etc. More recently, it is of increasing research interest to find a good antenna selection scheme that can significantly reduce the cost while incurring little performance loss. Generally, there are two goals for antenna subset selection in MIMO systems: One aims to maximize the channel capacity [3], [4], and the other aims to minimize the bit error rate (BER) for spatial multiplexing systems when some practical signaling schemes are used [5], [6], [7]. A transmit antenna selection algorithm is proposed based on minimum vector symbol error rate of multiplex system in [8]. When maximum likely receiver or linear receiver is used, the minimum singular value of channel is used to choose the optimal subset. However, these methods need a mass of search over all possible subsets, which have large complexity cost.

In this paper, we propose an algorithm based on complex Householder QR factorization avoiding exhaust search, for selecting a subset of transmit antennas. The proposed algorithm requires much lower communication complexity and its performance is as good as post-processing SNR method [8].

II. System Model

Assume a nonselective linear time invariant multiple element antennas channel between N_t transmit and N_r receive antennas. The input-output relationship for the system is given by [9]

$$\mathbf{r}(t)=\sqrt{\frac{E_s}{N_s}}\mathbf{Hs}(t)+\mathbf{n}(t) \tag{1}$$

where

$$\mathbf{s}(t)=\left[s_1(t),s_2(t),\cdots,s_{N_t}(t)\right]^T \tag{2}$$

$$\mathbf{r}(t)=\left[r_1(t),r_2(t),\cdots,r_{N_r}(t)\right]^T \tag{3}$$

$$\mathbf{n}(t)=\left[n_1(t),n_2(t),\cdots,n_{N_r}(t)\right]^T \tag{4}$$

are the transmitted signal, the received signal, and the zero-mean additive noise vectors, respectively, E_s is the average SNR, and $(\cdot)^T$ stands for transpose. Without loss of generality, it is assumed that $\mathrm{E}\left\{\left|n_i(t)\right|^2\right\}=1$ for all t and i , where $\mathrm{E}\{\cdot\}$ denotes the statistical expectation. $\mathbf{H}$ is the $N_r \times N_t$ channel matrix. When the channel is correlated, the model of channel can be described as follows

$$\mathbf{H}=\mathbf{R}_R^{1/2}\mathbf{H}_w\mathbf{R}_T^{1/2} \tag{5}$$

where $\mathbf{R}_T$ and $\mathbf{R}_R$ are the correlation matrix of transmit and receive antennas, $\mathbf{H}_w$ denotes the i.i.d. (independent, identically distributed) circular complex Gaussian random variables with variance 0.5 in each dimension.

We assume uniform linear arrays with the distance (in carrier wavelengths) between adjacent elements equal to

Δ_T and Δ_R at the transmitter and receiver, respectively. And we also assume that there are L scattering clusters with the angle of departure from the transmitter and angle of arrival at the receiver of the l th cluster denoted by $\theta_{T,l}$ and $\theta_{R,l}$, which are assumed to be Gaussian distributed with $\theta_{T,l} \sim N(\tilde{\theta}_{T,l}, \sigma^2_{T,l})$ and $\theta_{R,l} \sim N(\tilde{\theta}_{R,l}, \sigma^2_{R,l})$ respectively. For small spread angle (small σ_θ), the correlation matrix associated with the l th scattering cluster are given by

$$[\mathbf{R}_{T,l}]_{m,n} = \exp(-j2\pi(m-n)\Delta_T \cos(\tilde{\theta}_{T,l})) \times\exp(-\frac{1}{2}[2\pi(m-n)\Delta_T \sin(\tilde{\theta}_{T,l})\sigma_{T,l}]^2) \quad (6)$$

$$[\mathbf{R}_{R,l}]_{m,n} = \exp(-j2\pi(n-m)\Delta_R \cos(\tilde{\theta}_{R,l})) \times\exp(-\frac{1}{2}[2\pi(n-m)\Delta_R \sin(\tilde{\theta}_{R,l})\sigma_{R,l}]^2) \quad (7)$$

Under the narrowband assumption, the overall correlation matrices are the weighted averages of the correlation matrices associated with each scattering cluster, with the weights equal to the fraction of total power in each cluster [10].

III. ANTENNA SELECTION ALGORITHM

The linear receiver is the simplest spatial multiplexing receiver since it requires only a matrix multiply to separate the substreams. Therefore, it will be of interest in practical systems, particularly those with large numbers of transmit and receive antennas.

For linear receiver the post-processing SNR of the i th substream is given out in [8]

$$\text{SNR}_i = \frac{E_s \left|\mathbf{g}_i^T \mathbf{h}_i\right|^2}{N_s N_0 \left\|\mathbf{g}_i\right\|^2 + E_s \sum_{j\neq i} \left|\mathbf{g}_i^T \mathbf{h}_j\right|^2} \quad (8)$$

For MMSE receiver, $\mathbf{G} = [\mathbf{H}^H\mathbf{H} + N_0 / E_s \mathbf{I}_{N_s}]^{-1}\mathbf{H}^H$, $(\cdot)^H$ stands for Hermitian, the above formulation can be written as

$$\text{SNR}_i^{(\text{MMSE})} = \frac{E_s}{N_s N_0 [\mathbf{H}^H\mathbf{H} + N_0 / E_s \mathbf{I}_{N_s}]^{-1}_{i,i}} - 1 \quad (9)$$

For ZF receiver, $\mathbf{G} = (\mathbf{H}^H\mathbf{H})^{-1}\mathbf{H}^H$, the SNR_i formulation can be written as

$$\text{SNR}_i^{(\text{ZF})} = \frac{E_s}{N_s N_0 [\mathbf{H}^H\mathbf{H}]^{-1}_{i,i}} \quad (10)$$

The minimum post-processing SNR among all N_s data substreams determines the symbol error probability of spatial multiplexing system with fixed symbol constellation. Let $\text{SNR}_{\min} = \min \text{SNR}_i$, and from equation (9) and (10) we can obtain that in order to get $\text{SNR}_{\min}$, the largest value of $[\mathbf{H}^H\mathbf{H}]^{-1}_{i,i}$ should be written as small as possible.

$$[\mathbf{H}^H\mathbf{H}]^{-1}_{i,i} = \mathbf{e}_i^T[\mathbf{R}^H\mathbf{Q}^H\mathbf{Q}\mathbf{R}]^{-1}\mathbf{e}_i = \mathbf{e}_i^T[\mathbf{R}^H\mathbf{R}]^{-1}\mathbf{e}_i \quad (11)$$

where $\mathbf{e}_i$ denotes the i th column of identity matrix. Assuming $N_s = N_r$, the matrix $\mathbf{R}$ is square and upper triangular matrix, so $\mathbf{R}^{-1}(i,i)\times\mathbf{R}(i,i) = 1$. The formulation (11) can be rewritten as

$$\begin{aligned} \mathbf{e}_i^T\left[\mathbf{R}^H\mathbf{R}\right]^{-1}\mathbf{e}_i &= \mathbf{e}_i^T R^{-1}\left(\mathbf{R}^H\right)^{-1}\mathbf{e}_i = \left\|\mathbf{R}^{-1}(i,:)\right\|^2 \\ &\geq \left(\mathbf{R}^{-1}(i,i)\right)^2 = r_{i,i}^{-2} \end{aligned} \quad (12)$$

For V-BLAST detection, with the assumption of correct previous decisions, the post-processing SNR for the i th substream is given by

$$\text{SNR}_\text{i} = \frac{E_s}{N_s N_0}\left|r_{i,i}\right|^2 \quad (13)$$

From the formulations above, we can see that the post-processing SNR of ZF, MMSE and V-BLAST detection is proportional to the $\left|r_{i,i}\right|$ of diagonal element of $\mathbf{R}$.

We choose vector symbol error rate VSER as performance criterion which is given by [8].

$$\begin{aligned} P &= 1 - \prod_{i=1}^{N_s}(1-P_i) \\ &\leq 1-(1-P_{i\min})^{N_s} \\ &\approx N_s P_{i\min} \\ &\leq N_s N_e Q\left(\sqrt{\text{SNR}_{\min}\frac{d^2_{\min}}{2}}\right) \end{aligned} \quad (14)$$

where $d^2_{\min}$ is the square of minimum distance of symbol constellation of each transmit antenna. Assuming fixed constellation, the VSER is P_i corresponding to SNR_i. We can choose the subset with the largest $\text{SNR}_{\min}$ from all possible subsets, which is so-called antenna selection algorithm based on post-processing SNR. Its computation complexity is $O(N_r C_{N_t}^{N_s})$. In this paper, we propose an algorithm based on complex Householder QR factorization that is applicable for the wire MIMO systems. Its complexity is dramatically reduced compared with the algorithm based on post-processing SNR. And it has much lower flops than other QR algorithms [11]. The complex Householder transform is given by [12].

Let

$$\begin{aligned} x\mathbf{w} &= \mathbf{z} = \mathbf{a} - \mathbf{b} \neq \mathbf{0} \\ x &= \mathbf{w}^H\mathbf{a} + \mathbf{a}^H\mathbf{w} \\ x^2 &= \mathbf{z}^H\mathbf{a} + \mathbf{a}^H\mathbf{z} = \mathbf{z}^H\mathbf{z} \end{aligned} \quad (15)$$

where x is a positive real value, and $\mathbf{w}$ is a unit vector in C^n, i.e., $\left\|\mathbf{w}\right\| = 1$. We can derive the complex Householder transform

$$\mathbf{H} = \mathbf{I} - \frac{\mathbf{z}\mathbf{z}^H}{\mathbf{z}^H\mathbf{a}} \quad (16)$$

By complex Householder transform, we can get the QR factorization of channel $\mathbf{H} \in C^n$. Finally, according to the sort of $|r_{ii}|$, we can get the $\mathbf{H}_{N_s}$ that has higher SNR. The procedure and computation cost of the algorithm is given as follows

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(1) $\mathbf{A} = \mathbf{H}_{channel}$

(2) for $i = 1, ..., N_s$

(3) $\quad \mathbf{z}^{(i)} = \mathbf{a}^{(i)}{}_i - e^{(i)}{}_i \left\|\mathbf{a}^{(i)}{}_i\right\| \qquad o(N_r)$

(4) $\quad \mathbf{H}_i = \mathbf{I}^{(i)}{}_i - \dfrac{\mathbf{z}^{(i)}\mathbf{z}^{(i)H}}{\mathbf{z}^{(i)H}\mathbf{a}^{(i)}{}_i} \qquad o(N_r{}^2 N_s)$

(5) $\quad \mathbf{A}^{(i)} = \mathbf{H}^{(i)} \cdot \mathbf{A}^{(i-1)} \qquad o(N_r{}^2 N_s)$

(6) $\quad \mathbf{R}(i, i : N_t) = \mathbf{A}^{(i)}(1, 1 : N_t)$

(7) end

(8) $J_n := sort(r_{ii}^2) \downarrow; \bar{\mathbf{H}}_{channel} = [h_{j1}, \cdots, h_{jns}]$

Where $\mathbf{a}^{(i)}{}_i$ is i th column of $\mathbf{A}$, $\mathbf{I}^{(i)}{}_i$ is the identity matrix, $\|\cdot\|$ denotes the Frobinius norm, and J_n corresponds to the index of the column of $\mathbf{H}$ selected. The computation complexity of the proposed algorithm is $O(2N_t^2(N_r - N_t/3))$ flops [11], and the post-processing SNR method requires $O(N_r C_{N_t}^{N_s})$ flops. We can see that the proposed algorithm has grammatically reduced the computation complexity of selecting processing. And it's obvious that the complexity of the whole algorithm has much lower flops compared with the methods based on post-processing SNR.

IV. SIMULATION RESULTS

In this section, we present some simulations to justify the performance of the proposed algorithm. We consider a MIMO antenna selection system with $N_t = 4, N_r = 3$ and $N_s = 3$. Without loss of generality, in various linear receivers, ZF is used in the receive side. The performance is measured by VSER for a frame of 100 vector symbols from QPSK constellations averaged over 1000 frames.

In order to compare the performance of the proposed algorithm with other antenna selection techniques, we gives out the post-processing SNR method, the maximum capacity method, and no selection algorithm, and give out the performance upper bounds by ML receiver.

The first example, we consider a MIMO antenna selection system with the Rayleigh channel case where the elements of $\mathbf{H}$ are independently drawn from a complex zero-mean Gaussian distribution with the unit variance.

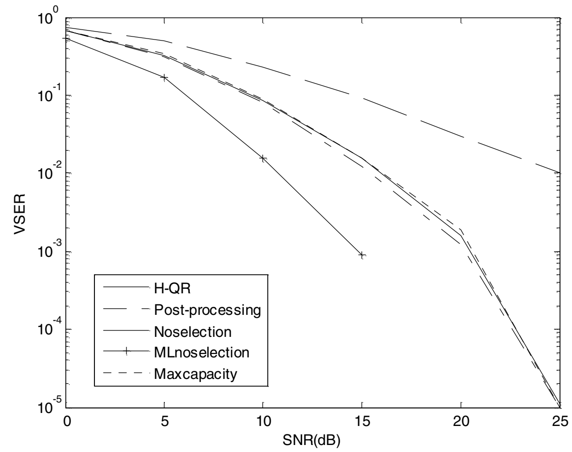

Figure 1. Performance comparison for different algorithms in uncorrelated channel

Fig. 1 shows the performance of VSER versus SNR in dependent channel. We can obtain that the proposed antenna selection algorithm dramatically improves the performance of the system compared with no selection. And in the same SNR, the VSER of the proposed method is very close to the post-processing SNR.

In the second example, we consider a correlated channel. The transmit antenna side has two equally weighted transmit clusters and both spread angles are small: $\bar{\theta}_{T,1} = 30^\circ$, $\bar{\theta}_{T,2} = 90^\circ$, $\sigma_{T,1} = 6^\circ$, $\sigma_{T,1} = 9^\circ$, $R_R = I_{N_S}$.

The Fig. 2 displays the VSER versus SNR in correlate channel. It shows an improved gain by selecting a subset of transmit antennas. Compared with post-processing method, the SNR loss of the proposed algorithm is 0.5 dB at same VSER.

In the third example, we give out the computation complexity of the complex Householder QR factorization algorithm and post-processing SNR method.

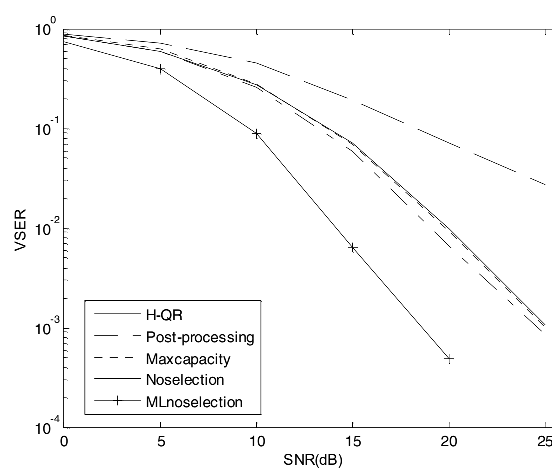

Figure 2. Performance comparison for different algorithms in correlated channel

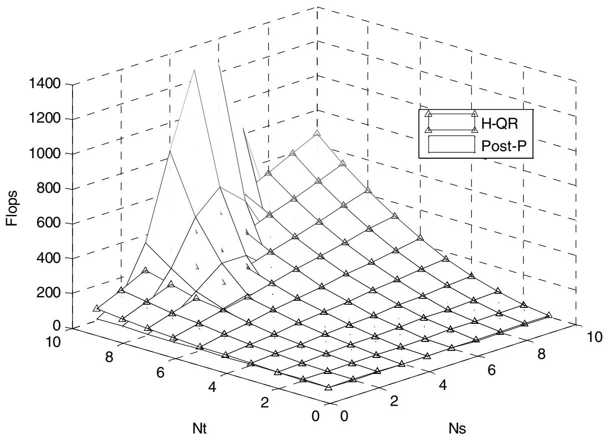

Figure 3. Flops versus the number of antenna

The Fig. 3 shows the growth trend of flops versus antenna number. It presents that the post-processing SNR algorithm requires much more flops. Especially when the number of transmit antenna is large, the post-processing complexity increased dramatically. And the proposed algorithm requires much fewer flops than the post-processing SNR.

V. Conclusion

In this paper, an antenna selection algorithm based on complex Householder QR factorization has been proposed in wireless MIMO systems. The proposed algorithm avoids exhaust search over all possible subsets of antenna, has much lower complexity cost, makes VSER close to the post-processing SNR method, and enjoys significantly better performance as compared with other algorithms. Simulation results demonstrate that in independent and correlated channel the proposed selection algorithm achieves the performance as excellent as the post-processing SNR method.

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